To Fred Mintz, Thanks for your interest in this talk. Allan

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Molecular relaxation effects on sonic boom waveforms: A tutorial survey

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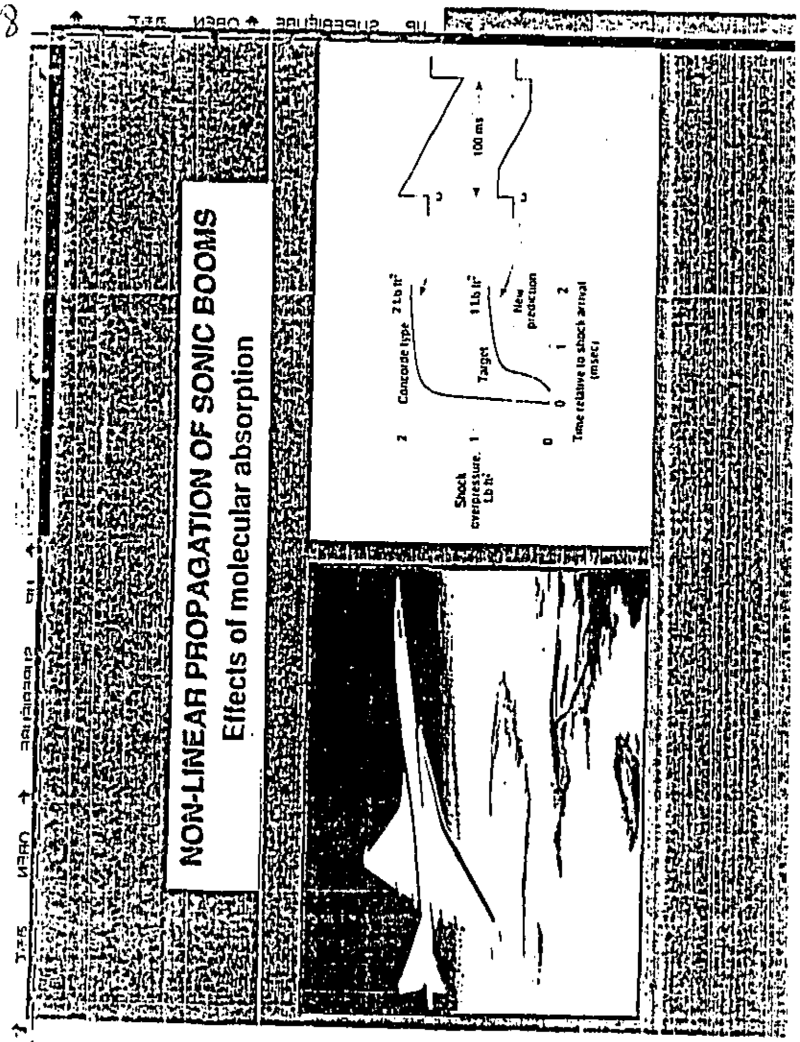
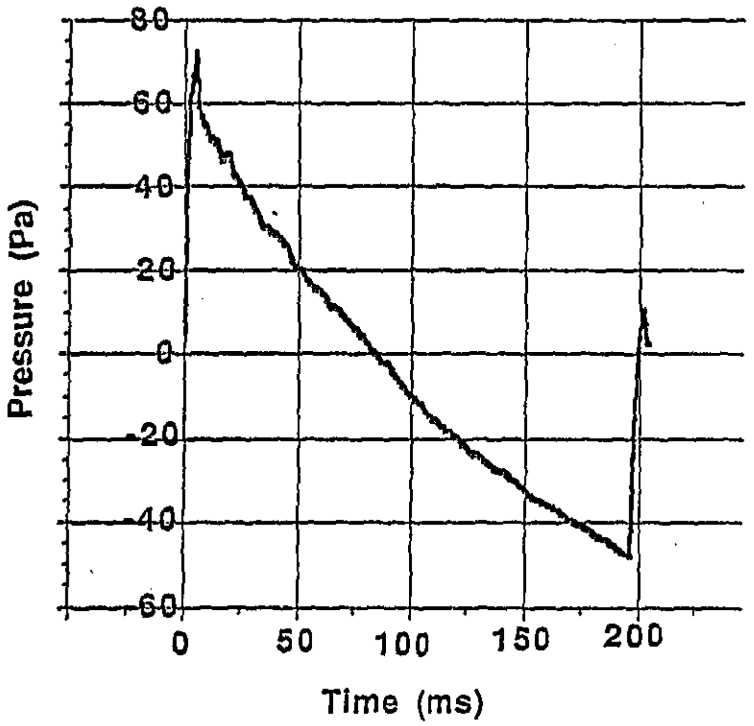
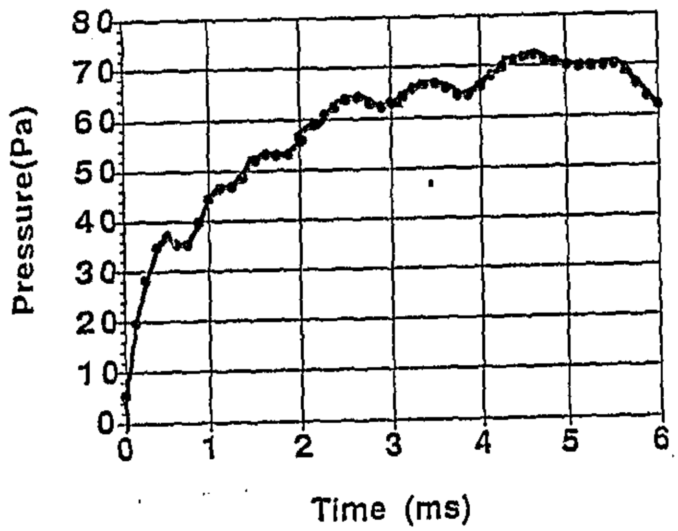


Sonic boom - example of spiked signature



Rise phase of a sonic boom (leading shock in the N-wave)



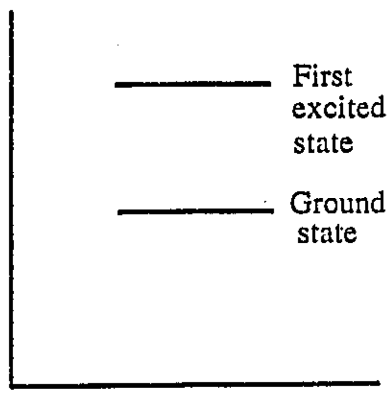
(This is the early portion of a spiked-signature waveform.)

SR-71 at Mach 2.6; Flight altitude is 66,000 ft

Flying over Mojave desert on August 5, 1987, 9:00 a.m.

What is molecular relaxation?

Nitrogen molecule



$$\frac{\text{Number in excited state}}{\text{Number in ground state}} = \text{function of temperature}$$

But this is so only for thermodynamic equilibrium.

Chronology

Taylor (Proc Roy Soc, 1910)
Becker (Zeit Phys, 1922)
Herzfeld and Rice (Phys Rev, 1929)
Knudsen (JASA, 1931, 1933)
Tisza (Phys Rev, 1942)
Lagerstrom, Cole, and Trilling (Cal Tech, 1949)
Markham, Beyer, and Lindsay (Rev Mod Phys, 1951)
Truesdell (J Rat Mech Anal, 1953)
Mendousse (JASA, 1953)
Lighthill (Batchelor, 1956)
Hayes (Emmons, 1958)
Polyakova, Soluyan, and Khoklov (Sov. Phys. Acoust., 1962)
Lilley (Liege ICA, 1965)
Clarke and Rodgers (JFM, 1965)
Piercy (JASA, 1969)
Ockendon and Spence (JFM, 1969)
Hodgson and Johannesen (JFM, 1971)
Hodgson (JFM, 1973)
Ffowcs-Williams and Howe (JFM, 1973)
Pestorius (UT-ART, 1973)
Anderson (UT-ART, 1974)
Tubb (AIAA, 1975)
Pierce (JSV, 1978)
Bass and Raspet (JASA, 1978)
Johannesen and Hodgson (Rep Prog Phys, 1979)
Morfey (AGARD, 1979)
Crighton and Scott (Phil Trans, 1979)
Pierce (Acoustics, 1981)
Holst-Jensen (UTIAS, 1981)
Honma, Glass, Holst-Jensen, and Tsumita (Shock wave symposium, 1981)
Orenstein (UT-ARL, 1982)
Bass, Ezell, and Raspet (JASA, 1983)
Honma and Glass (Proc Roy Soc, 1984)
Lardner and Nicklason (Wave Motion, 1986)
Bass, Layton, Bolen, and Raspet (JASA, 1987)
Kang and Pierce (Auburn, 1990)
Pierce and Kang (ISNA, 1990)
Raspet and Bass (AIAA, 1990)
Kang (PSU, 1991)
Raspet, Bass, Yao, and Wu (NASA, 1992)

Parameters characterizing a relaxation process:

- a relaxation time

$$\tau_{\text{relax}}$$

- a sound speed increment

$$\Delta c = c_{\text{prop,froz}} - c_{\text{prop,eq}}$$

Two relaxation processes for air:

- Vibrational relaxation of oxygen molecules
- Vibrational relaxation of nitrogen molecules

ΔE = Quantum energy gap between ground and first excited vibrational state

k = Boltzmann's constant

When gas is in equilibrium:

Average kinetic energy per molecule (translational plus rotational) $= \frac{5}{2}kT_{\text{eq}}$

Fraction of molecules in first excited vibrational state $= e^{-\Delta E/kT_{\text{eq}}}$

For gas not in equilibrium, define apparent temperatures $T_{\text{tr,rot}}$ and T_{vib} such that

Average kinetic energy per molecule (translational plus rotational) $= \frac{5}{2}kT_{\text{tr,rot}}$

Fraction of molecules in first excited vibrational state $= e^{-\Delta E/kT_{\text{vib}}}$

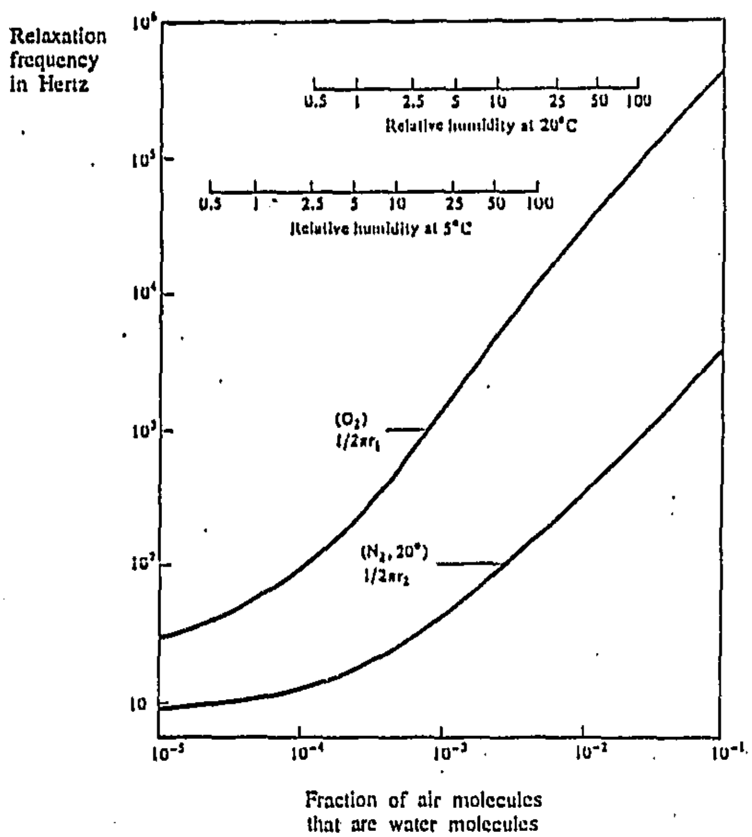
Relaxation equation:

$$\frac{dT_{\text{vib}}}{dt} = \frac{1}{\tau_{\text{relax}}}\left(T_{\text{tr,rot}} - T_{\text{vib}}\right)$$

$$\left\{\frac{d}{dt} + \frac{1}{\tau_{\text{relax}}}\right\}\left(T_{\text{vib}} - T_{\text{tr,rot}}\right) = -\frac{d}{dt}T_{\text{tr,rot}}$$

Relaxation times very sensitive to humidity!

Very low humidity means
very long relaxation times.



internal energy per unit mass

$$\mathcal{E}_{\text{int}} = \mathcal{E}_{\text{tr,rot}} + \mathcal{E}_{\text{vib}}$$

---

limiting cases for
specific heat at constant volume:

equilibrium: $$c_{v,\text{eq}} = \frac{d\mathcal{E}_{\text{tr,rot}}}{dT_{\text{eq}}} + \frac{d\mathcal{E}_{\text{vib}}}{dT_{\text{eq}}}$$

frozen: $$c_{v,\text{frozen}} = \frac{d\mathcal{E}_{\text{tr,rot}}}{dT_{\text{tr,rot}}}$$

---

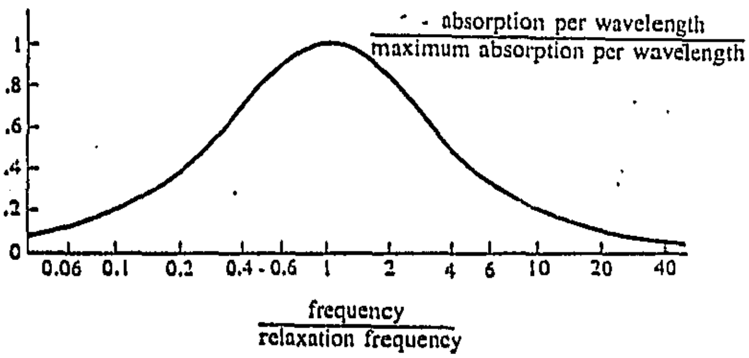
specific heat ratio: $$\gamma = \frac{c_p}{c_v} = 1 + \frac{R}{c_v}$$

sound speed: $$c_{\text{prop}} = \left[\gamma RT\right]^{1/2}$$

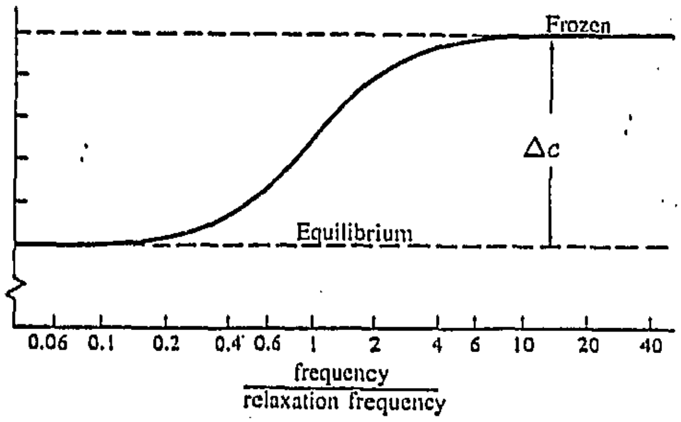
$$c_{\text{prop,froz}} = c_{\text{prop,eq}} + \Delta c$$

$\Delta c =$ sound speed increment
associated with a relaxation process

Frequency dependence of a single relaxation process

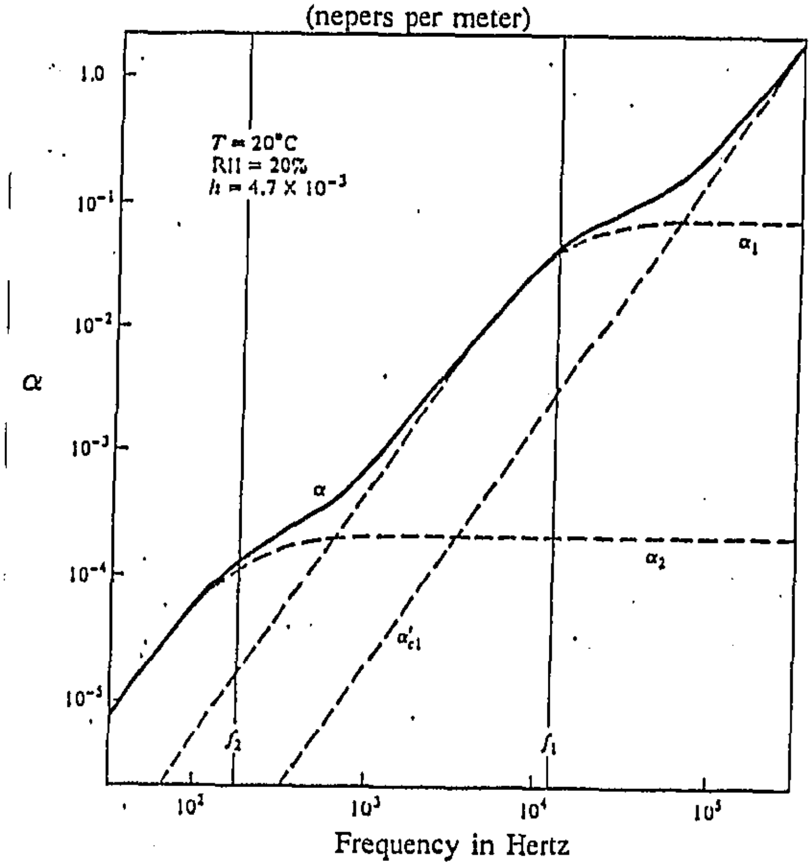


ıase velocity



Atmospheric propagation:

Absorption coefficient α
(nepers per meter)



One-way propagation equation for a dissipative medium (Burgers' equation)

$$\frac{\partial p}{\partial t} + c\frac{\partial p}{\partial x} + \frac{\beta p}{\rho c}\frac{\partial p}{\partial x} - \delta\frac{\partial^2 p}{\partial x^2} = 0$$

where

$$2\rho\delta = \frac{4}{3}\mu + \mu_{\text{bulk}} + (\gamma - 1)\frac{\kappa}{c_p}$$

If you neglect nonlinear term, assume t and x dependence as

$$e^{-i\omega t}e^{i(\omega/v_{\text{ph}})x}e^{-\alpha x}$$

you get classical absorption coefficient

$$\alpha_{\text{cl}} = \frac{\omega^2}{c^3}\delta$$

Excerpt (slightly altered) from
The Structure of Shock Waves at Large Distances from Bodies Travelling at High Speeds
[G. M. Lilley, 5th ICA, Liege, Belgium, 1965]

The Taylor rise time τ_{rise} of the shock wave is given as the time over which 0.9 of the overall jump occurs. Thus we find

$$\tau_{\text{rise}} = \frac{6\rho_0\delta}{P_{\text{sh}}} \quad \left\{\text{corrected to } \frac{9.8\rho_0\delta}{P_{\text{sh}}}\right\}$$

where $\quad 2\rho_0\delta = \frac{4}{3}\mu + \mu_{\text{bulk}} + (\gamma - 1)\frac{\kappa}{c_p}$.

In estimating diffusivity δ it is necessary to know the value of the bulk viscosity accurately. Lighthill (*Surveys in Mechanics* article, 1956) has argued that the presence of minute traces of water vapour in air *effects* the vibrational energy exchange between the oxygen and water vapour molecules and this results in exceedingly large values of μ_{bulk}. But for perfectly dry air $\mu_{\text{bulk}} \approx \mu$. *(?????)* Thus depending on the value of μ_{bulk} we find the shock rise time can vary from 4 μs to 40 μs for a weak shock wave of 1 lb/ft^2 (50 Pa) pressure jump.

Tisza's observation (paraphrased)

Physical Review, 1942

At sufficiently low frequencies,
the effect of any given relaxation process

is equivalent to
what results from increasing the bulk viscosity by

$$\Delta\mu_{\text{bulk}} = \left\{2\rho c\right\}\left\{\tau_{\text{relax}}\right\}\left\{\Delta c\right\}$$

Perfectly dry air at low frequencies will have a very large bulk viscosity!

Linear dispersion relation with relaxation included

$$\underset{\text{big}}{ck} = \underset{\text{big}}{\omega} + \underset{\text{small}}{i\frac{\omega^2}{c^2}\delta} + \frac{\omega}{c}\sum_{\nu}\underset{\text{small}}{(\Delta c)_\nu}\frac{i\omega\tau_\nu}{1-i\omega\tau_\nu}$$

$$\underset{\text{big}}{\omega} = \underset{\text{big}}{[c]k} - \underset{\text{small}}{ik^2\delta} - k\sum_{\nu}\underset{\text{small}}{(\Delta c)_\nu}\frac{ikc\tau_\nu}{1-ikc\tau_\nu}$$

How to derive transient wave equation
from dispersion relation

$$-i\omega \to \frac{\partial}{\partial t}; \qquad ik \to \frac{\partial}{\partial x}$$

Introduce internal auxiliary variables, p_1 and p_2:

$$\frac{ikc\tau_\nu}{1-ikc\tau_\nu}p \to -p_\nu$$

$$c\tau_\nu\frac{\partial p}{\partial x} = -p_\nu + c\tau_\nu\frac{\partial p_\nu}{\partial x}; \qquad \tau_\nu\frac{\partial p}{\partial t} = p_\nu + \tau_\nu\frac{\partial p_\nu}{\partial t}$$

Nonlinear correction (Whitham's rule):

$$[c] \to c + v + \frac{dc}{dp}p = c + \frac{\beta p}{\rho c}$$

One way nonlinear propagation system with relaxation included

a generalization of Burgers' equation

$$\underbrace{\frac{\partial p}{\partial t}}_{\text{big}} + \left[\underbrace{c}_{\text{big}} + \underbrace{\frac{\beta p}{\rho c}}_{\text{small}}\right]\frac{\partial p}{\partial x} - \underbrace{\delta\frac{\partial^2 p}{\partial x^2}}_{\text{small}} + \sum_\nu \underbrace{(\Delta c)_\nu \frac{\partial p_\nu}{\partial x}}_{\text{small}} = 0$$

supplemented by relaxation equations ($\nu = 1,\ 2$)

$$c\tau_\nu \frac{\partial p}{\partial x} = -p_\nu + c\tau_\nu \frac{\partial p_\nu}{\partial x}$$

or

$$\tau_\nu \frac{\partial p}{\partial t} = p_\nu + \tau_\nu \frac{\partial p_\nu}{\partial t}$$

Molecular relaxation incorporated into sonic boom waveform predictions

Burgers' equation with added molecular relaxation term:

$$\frac{\partial p}{\partial t} + c\frac{\partial p}{\partial x} + \underbrace{NST}_{\text{nonlinear steepening term}} + \underbrace{TVT}_{\text{classical absorption term}} + \underbrace{MRT}_{\text{molecular relaxation term}} = 0$$

coupled with relaxation equations:

$$p_\nu + \tau_\nu\frac{\partial p_\nu}{\partial t} = \tau_\nu\frac{\partial p}{\partial t}$$

$$\nu = O_2, N_2$$

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Similitude Solution
for waveform in vicinity of shockfront
(in tradition of G. I. Taylor and R. Becker)

primary (defendable somewhat) assumption:

$$p(x,t) = F(x - V_{sh}t); \quad p_\nu(x,t) = F_\nu(x - V_{sh}t)$$

reduces coupled pde's to coupled ode's

Nominal shock location where $x - V_{sh}t = 0$

To describe shock profile it is sufficient to seek solution corresponding to a net jump:

$F(\xi) \to 0$ as $\xi \to \infty$

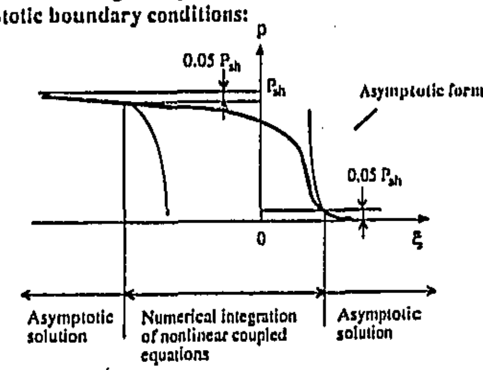
$F(\xi) \to P_{sh}$ as $\xi \to -\infty$

Complete set of boundary conditions to pin down the solution of the three coupled ode's requires a nontrivial derivation.

Shock speed V_{sh} emerges as part of the solution.

<u>Early portion of sonic boom waveform</u>

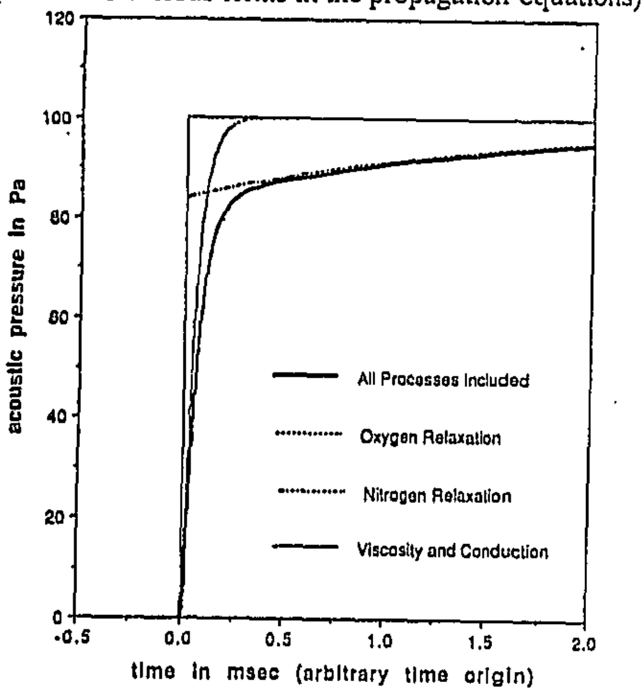
Theoretical predictions based on numerical solution of augmented Burgers' equation set matched to asymptotic boundary conditions:



Middle rise phase: O_2 relaxation dominates
Later rise phase: N_2 relaxation dominates

Early history of shock waveform:

(effects of various terms in the propagation equations)

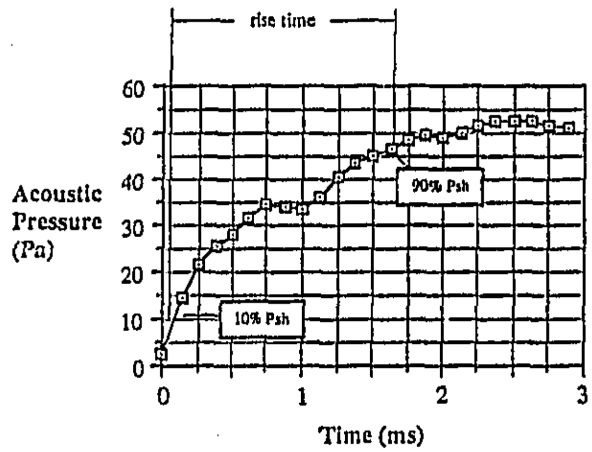


Computed results for when

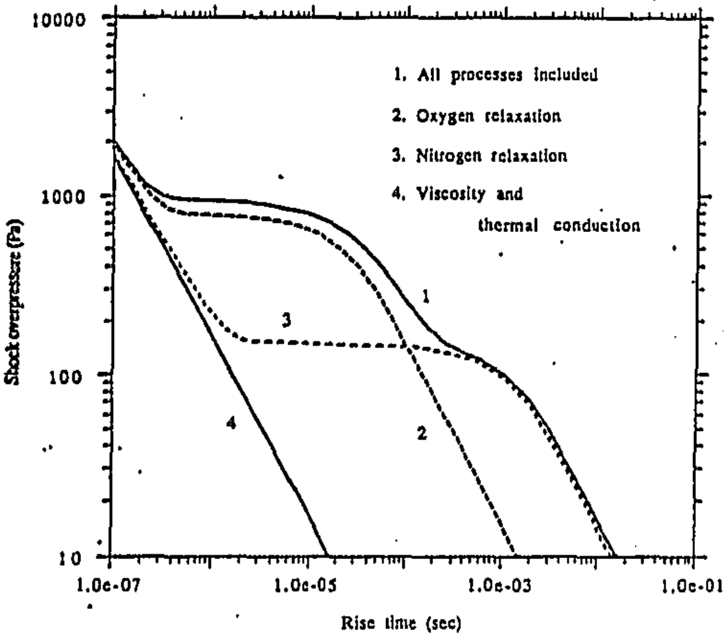
Net pressure jump is 100 Pa
Temperature is 20° C
Relative humidity is 10%

Definition of rise time as used here

- based on steady state shock overpressure "Psh"
- the time for pressure to jump from 10% to 90% of Psh



Psh is not always = maximum overpressure



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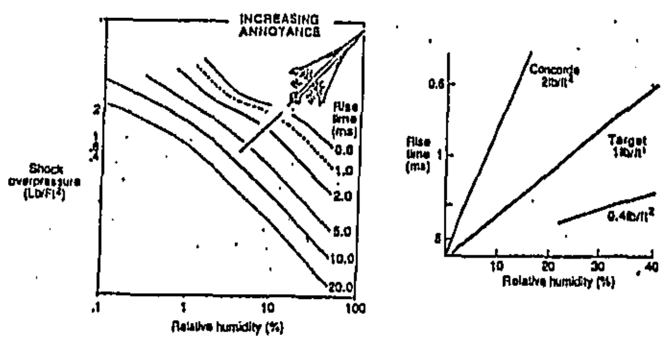
HUMIDITY EFFECTS ON SONIC BOOM WAVE FORMS



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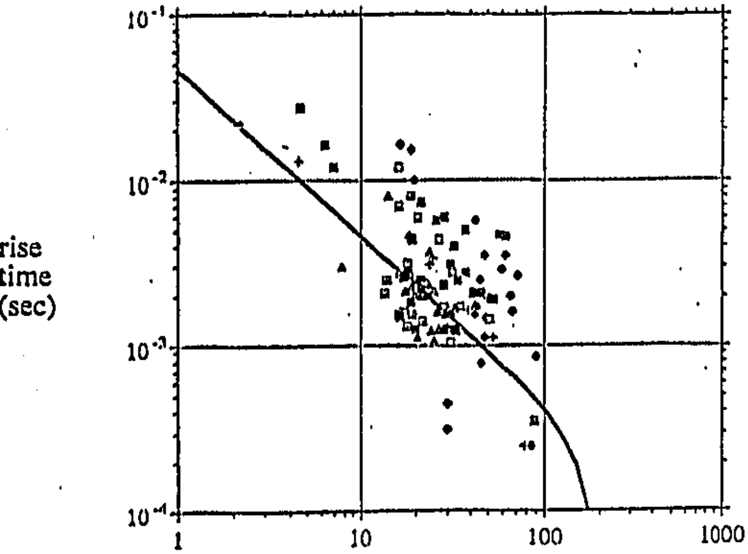
Rise times of actual sonic boom waveforms recorded at the ground

(Mojave desert, various Mach numbers, flight altitudes, airplanes)
(rel hum ≈ 24%, T ≈ 33° C)



Solid line is theoretical prediction
based on assumption that
pressure of incident waveform is one-half of that
measured at the ground.

Concluding Remarks

- Relaxation theory predicts rise times of right order of magnitude.
- Theoretical predictions of rise times tend to be lower than observed in field data.
- Strong dependence of relaxation theory rise times on humidity.
  Dry air leads to the longest rise times.
- For booms generated by next generation of civilian supersonic aircraft,
  nitrogen relaxation effects will be
  much more important than
  oxygen relaxation effects
- Rapidity with which waveform profiles adjust to changes in humidity along flight path is topic for further study