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UNITED STATES DEPARTMENT OF THE INTERIOR  
BUREAU OF MINES

SEISMIC EFFECTS OF QUARRY  
BLASTING

BULLETIN 442

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UNITED STATES DEPARTMENT OF THE INTERIOR

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BUREAU OF MINES

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Bulletin 442  
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## SEISMIC EFFECTS OF QUARRY BLASTING

BY

J. R. THOENEN and S. L. WINDES



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## SEISMIC EFFECTS OF QUARRY BLASTING<sup>1</sup>

By J. R. THOENEN<sup>2</sup> AND S. L. WINDES<sup>3</sup>

### INTRODUCTION

The proximity of residential districts to quarry operations has long caused concern to quarry operators and civic organizations charged with responsibility for municipal expansion and safety. In many instances, litigation has been instituted in which the resident claimed that vibrations transmitted through the ground from blasts in the quarry caused damage to his home. Until recently there was no accurate means for measuring such seismic vibrations and evaluating their damaging effect, if any, on adjacent structures. At the request of the quarry industry, the Bureau of Mines undertook research to ascertain the physical characteristics of seismic disturbances from blasting in quarries and to evaluate their effect on typical structures.

The research program on seismic disturbances was completed in the fall of 1940. From time to time during the study current results were published in progress reports to inform the interested public of the facts as they were accumulated. The study itself involved highly technical and scientific elements, and in order that the lay reader would not be confused by technical phraseology it was frequently necessary to state conclusions without elaborating on the detailed steps leading to them. As a result, the progress reports were probably open to criticism from the technician.

Early research uncovered many characteristics and phenomena that apparently were contradictory, and until the study was complete their proper interpretation frequently was obscure. This was another reason for omitting technical details until all phases of the problem had received sufficient investigation to clarify certain aspects.

### OBJECT OF BULLETIN

The object of this bulletin is to review briefly the previous publications so that the reader may follow the progress of the research and to supply the technical details supporting the conclusions reached.

In any such study, evolution of the technique invariably uncovers many related avenues of research which are of scientific interest but have only indirect bearing on the main problem. Related problems, while fully recognized from time to time, have been deliberately omitted from the study in order that a solution of the main problem

<sup>1</sup> Work on manuscript completed March 19, 1941.

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would not be delayed unduly. Although related subjects offer extremely interesting leads for scientific study, they are not considered to be within the province of the Bureau in its study of practical problems of mine operation.

#### HISTORY OF STUDY

In March 1930 correspondence was begun between Dr. Charles E. Munroe, of the Bureau of Mines, and executives of the quarry and explosives-manufacturing industries with the object of initiating a major research into ways and means of settling what damage, if any, was caused by seismic vibrations from quarry blasts. Early in 1930 a geophysical section was formed in the Bureau with Dr. F. W. Lee as the supervising engineer. Dr. Munroe recognized the possibility of utilizing geophysical equipment for measuring and recording earth vibrations and turned the problem over to Dr. Lee.

A cooperative agreement was reached between the Bureau and Otho M. Graves, president of the General Crushed Stone Co., Easton, Pa., representing his own and six other operating companies.<sup>4</sup> A small initial Bureau appropriation was obtained, which was supplemented by a fund raised by the seven companies.

Work was begun under the direction of Dr. Lee to develop suitable equipment for recording the ground movement caused by the seismic wave set up by a quarry blast. This work was conducted at Bucknell University, Lewisburg, Pa., and at the Central Experiment Station of the Bureau at Pittsburgh, Pa., under the personal direction of Dr. George A. Irland. Development progressed slowly because of inadequate funds and personnel for continuous study.

A preliminary survey of available equipment disclosed the fact that no provision had been made for accurately calibrating the seismographs or seismometers already in use. As a consequence, considerable confusion existed as to just what the instruments actually recorded, that is, whether velocity or acceleration of ground movement or actual displacement. It was demonstrated that some instruments when once set in motion vibrated at their own natural frequency. Seismographs used for recording vibrations from earthquakes commonly recorded acceleration of ground movement. To convert these records to displacement involved a possible mathematical error of as much as 100 percent, according to Dyk (14).<sup>5</sup> Dr. Lee felt that instruments for this work should be designed and constructed so as to record ground displacement directly without mathematical calculations. To accomplish this it was necessary to design and build calibrating tables that could be made to vibrate accurately at known frequencies and amplitudes at the will of the operator. Furthermore, these tables had to be designed to maintain the frequency constant while the amplitude was varied at will within the range desired or to hold the amplitude constant while the frequency was varied within the desired range. These features were necessary in order that the accuracy of the seismometers could be checked (27). The calibration tables were constructed at Pittsburgh and installed at Bucknell

<sup>4</sup> The seven original companies were General Crushed Stone Co., Easton, Pa.; New Haven Trap Rock Co., New Haven, Conn.; West Roxbury Trap Rock Co., West Roxbury, Mass.; Massachusetts Broken Stone Co., Waltham, Mass.; Lynn Sand & Stone Co., Swampscott, Mass.; J. S. Lane & Son, Inc., Meriden, Conn.; and Rowe Contracting Co., Stoughton, Mass.

<sup>5</sup> Italicized numbers in parentheses refer to references in the bibliography.

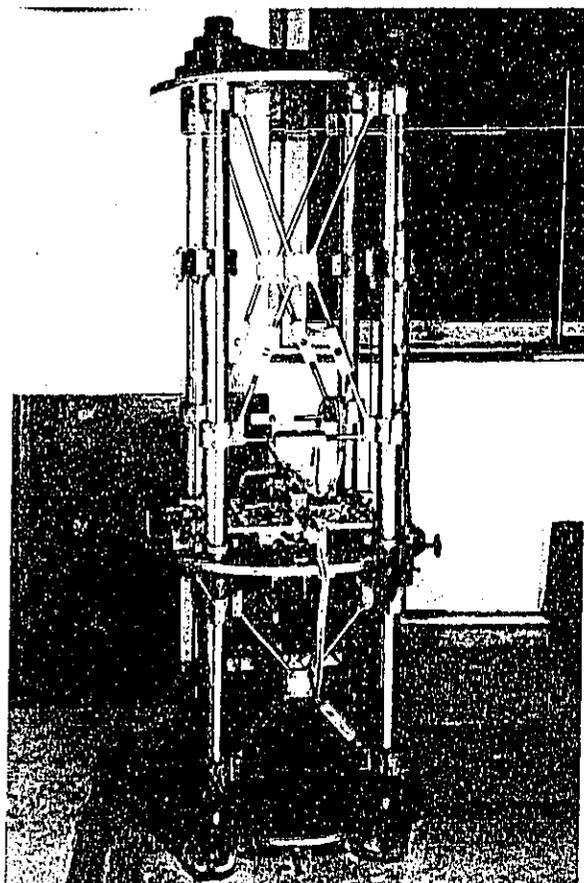


FIGURE 1.—Seismometer calibration table.

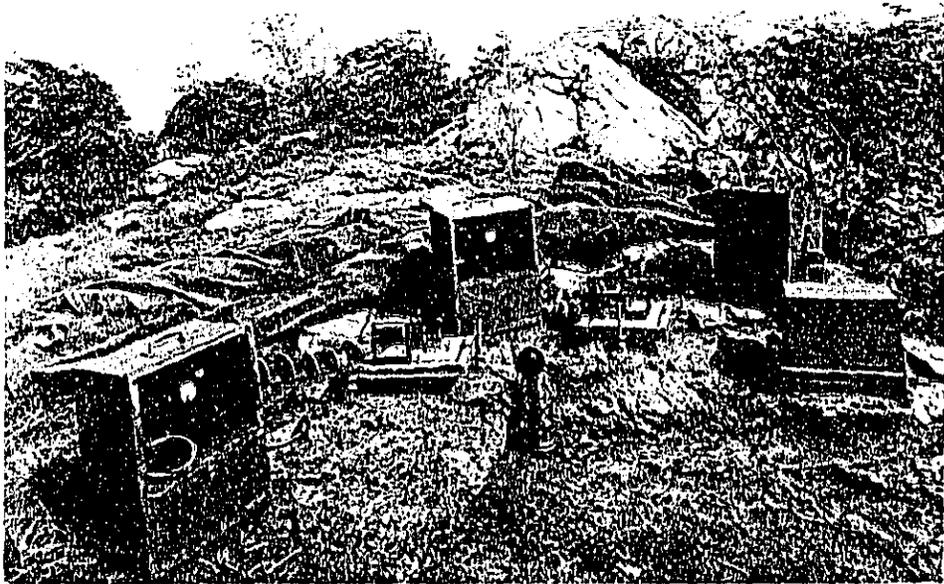


FIGURE 2.—Selammeter station.

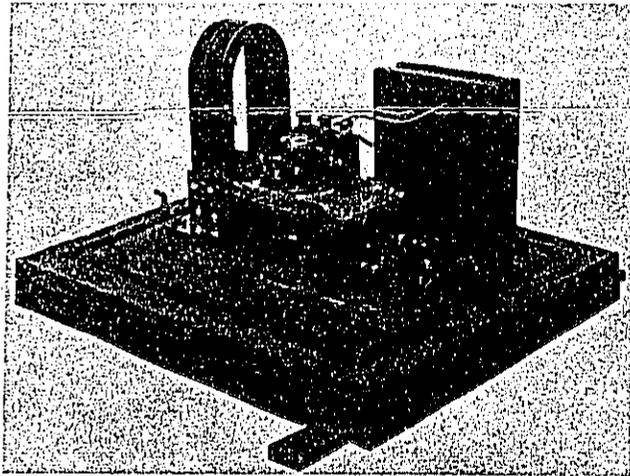


FIGURE 3.—Vertical seismometer.

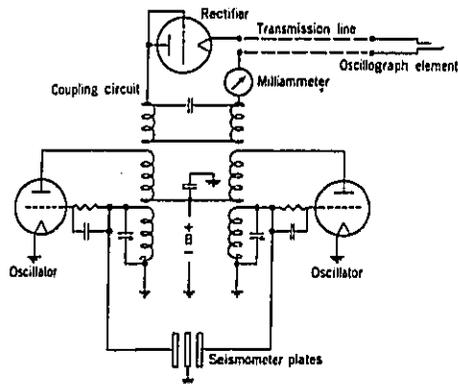


FIGURE 4.—Seismometer electrical circuit.

University. Later they were moved to the laboratory of the Eastern Experiment Station of the Bureau at College Park, Md. (See fig 1.)

The next step in the program was the construction of suitable seismometers. This step involved many intricate technical problems. As actual ground displacement or movement was sought, it was convenient to use three pick-up devices or seismometers at each recording point. (See fig 2.) Two recorded the horizontal components of motion at right angles and the third the vertical component; from these records, the actual movement both in magnitude and direction could be computed mathematically without appreciable error. Certain features of design were necessary. For use in the field the instruments must be portable, and they must be easy to adjust to a rather wide range of sensitivity so that readable records could be obtained from blasts of different magnitudes or the same magnitude at different distances.

The impulses picked up by the seismometers must be transmitted accurately to a central point and simultaneously recorded on a moving sheet of photographic paper. For this purpose electrical transmission and the Duddell type of oscillograph were utilized. The details of design of both the seismometers and the transmitting and recording equipment have been discussed in Bureau publications (18, 27).

In its finished design, the seismometer consists of a hinged middle plate swinging between two parallel outer plates, all electrically insulated from one another. (See fig. 3.) The two outer plates are rigidly connected to the frame of the seismometer, but the center plate is suspended to move freely between them. The two outer plates thus follow the vibration or movement of the ground on which the seismometer rests, and as they move the distance between one outer plate and the center plate increases, whereas the distance between the other outer plate and the center plate decreases. As a result the electrical capacity between the middle and each outer plate varies in proportion to the ground movement. These differences in capacity are converted into differences in electric current by means of a vacuum-tube circuit (fig. 4) similar in many respects to the common radio-receiving set. The plates of the seismometer act as two radio condensers. These condensers control the frequency of two vacuum tubes or oscillators. Thus, movement of the outer plates changes the frequency of the oscillators in opposite directions. The two oscillators are coupled and adjusted to synchronize. Movement of the seismometer plates also tends to increase or decrease the coupling current between the oscillators. This change in current is proportional to the ground movement, but as the current is a high-frequency alternating current it is unsatisfactory for use in the oscillograph for recording purposes. It is converted to direct current by a third vacuum tube or rectifier. The direct current is then sent over a connecting wire to the oscillograph element and actuates a light beam, whose movement can be photographed. The oscillograph contains 12 elements so that records can be made simultaneously from 12 seismometers. It also has a timing-fork arrangement by means of which vertical lines are projected on the moving photographic paper at intervals of one-hundredth second. The width between these timing lines is regulated by the speed of the drum driving the paper.

By 1935 suitable calibrating equipment, seismometers, and field equipment had been built and tested for use. In June of that year

Dr. Lee and the geophysical prospecting studies of the Bureau were transferred to the Geological Survey; however, the study of seismic vibrations from quarry blasting was considered outside the field of geophysical prospecting, as conducted by the Survey, and was retained by the Bureau. In July 1935 funds were allotted to the non-metal mining section for study of this problem. A field party was organized, and preliminary tests were made on the surface of the ground over the Bureau's Experimental coal mine at Bruceston, Pa. The immediate object of the tests was to familiarize the inexperienced field party with the equipment and to perfect a field technique. During 1936 the field party spent 8 months visiting quarries throughout the South and East, observing and recording ground movements from quarry shots. This study involved the cooperation of the operators of many quarries in addition to the original seven. It brought to light some mechanical defects in the equipment and indicated various ways to perfect technique. Considerable fundamental information was accumulated.

In the spring of 1937, an opportunity arose for continuing the tests in a mine in which many troublesome conflicting factors incident to quarry blasting could be eliminated or controlled so that they could be accurately evaluated. These underground tests, conducted over a period of several months, added to the valuable fundamental data that could be applied to future quarry tests.

Prior to this time all tests were directed toward ascertaining the characteristics of the vibrations under different conditions of stratigraphy of the transmitting mediums, source of excitation, distance from the source, and size of the explosive charge causing the seismic wave.

In the fall of 1937, tests were made in a house directly above the Bureau's testing adit at Mount Weather, Va. Progressively larger explosive charges were set off until actual damage occurred in the house, and the results were recorded.

In spite of a diligent search no other structures that could be tested to the damage point by actual explosive blasts were found, consequently it was necessary to test by mechanical vibration. A vibrator or shaker was obtained, and through the cooperation of another group of quarry operators various types of residential structures were found for test purposes. In these structures, permission was given to continue the tests to and beyond the damage point.

During 1938 and 1939 many houses were tested, and in several houses it was possible to compare the effects of quarry blasts with the effect of the mechanical shaker. In addition, numerous quarry tests were made to confirm or refute results obtained previously in which the evidence was not convincing.

In 1940 all the assembled data were reviewed and the results and conclusions reduced to mathematical expressions insofar as possible with a subject that cannot be classed as an exact science.

#### ACKNOWLEDGMENTS

The authors are indebted to the original seven companies whose interest and initial financial assistance made it possible to begin the study; to Dr. F. W. Lee for his continued interest and many helpful comments as the work progressed; to the many operators who per-

mitted the use of their quarries for experimental purposes, supplied labor and equipment for special tests, and furnished complete houses for damage tests at their own expense; and to the explosives manufacturers who furnished various commercial explosives and even especially prepared explosives for special tests.

The authors wish to commend Dr. F. W. Lee, who supervised the design and development of the seismometers and calibrating tables and organized the preliminary field work in which Dr. J. H. Swartz and Dr. J. W. Joyce conducted the early calibration tests; Dr. George A. Irland, professor of electrical engineering at Bucknell University, who developed the seismometers, built the first models, and assisted in subsequent calibration studies; and all the past and present members of the field party whose sincere interest and active and willing cooperation made the study a success. These members include James M. Dobbie who, with the junior author under the field supervision of Dr. Irland, made the first mine and quarry tests and developed the original field technique in 1935; August Raspel, Orin P. Gard, and Alan C. Byers, who assisted in the early office and field work on quarry tests; Abraham Yanovsky, who assisted in the underground mine tests, made a detailed study of wave shapes, and formulated rules for identifying wave shapes with certain mine localities; Philip Krupen, who worked out the first graphic solution for the index of damage from preliminary data; A. T. Ireland, who suggested tests that resulted in the first damage-index determination; and Jack T. Donovan, who has given invaluable assistance in the later stages of the study.

#### PREVIOUS PUBLICATIONS

During this investigation, several papers have been published dealing with particular phases of the work or with the progress of the general program. Of these, five have been published by the Bureau of Mines as reports of investigations that were essentially progress reports. One paper, published by the American Institute of Mining and Metallurgical Engineers, presented the results of some special tests to determine the mode of response of a house to both internal and external sources of vibrations. A seventh paper was read by the senior author before the Twenty-first Annual Convention of the National Crushed Stone Association at Cincinnati, Ohio, on January 24, 1938, and published in the Crushed Stone Journal for March-April 1938. This paper reviewed for quarry operators the progress of the tests up to that time. An eighth paper was presented before the American Geophysical Union, Division of the National Research Council, in April 1938 and published in its transactions for that year. It reviewed the phenomena of seismic motion as observed from quarry blasts.

In the first publication (28), issued in November 1936, the authors discussed known information on seismic vibrations gathered from the existing literature, advanced tentative theories, and suggested mathematical formulas for evaluating the relation of ground displacement to velocity, acceleration, and rate of change of acceleration. Although merely suggestions, these had a basis of justification in tests conducted at the Bureau coal mine at Bruceton, at a limestone quarry in Alabama, and at two quarries in South Carolina, one in granite and one in limestone.

For comparison, several tests were made to record the vibrational effect of passing automobiles, trucks, passenger trains, and other sources of agitation.

A test in a South Carolina limestone quarry illustrates the contradictory results obtained before field technique was perfected. Two shots were recorded in which the weights of the explosive charges were in the ratio of about 1:8. Records made at the same points for both shots showed the smaller charge gave the greater displacement at all seismometer stations. In the South Carolina granite quarry and the Alabama limestone quarry, seismometer stations farthest away gave larger displacement than those closer to the shots.

Both results were counter to expectations and remained unexplained until further experience and better technique showed the cumulative effects of various factors unsuspected at the time the tests were made.

Reference was made to the modified Mercalli scale, as developed by Harry O. Wood and Frank Neumann (26), to evaluate the effect of earthquake vibrations, and it was shown that vibrations from quarry blasts, if computed upon the same basis as earthquake vibrations, would appear as severe and damaging shocks on the Mercalli scale when in reality they were hardly noticeable.<sup>9</sup>

No definite conclusions were reached at this stage of the study, but a working field technique was established and considerable information accumulated on what not to do in field tests.

The second publication (43), issued a year later (November 1937), summarized the results observed in field work from January to August of that year and recorded in detail the effects of 61 blasts in 19 quarries in 6 Southern and New England States under regular operating conditions.

The types of shots ranged from mud caps through hammer, wagon, and churn-drill holes to large tunnel or coyote shots. The quantity of explosive ranged from  $\frac{1}{2}$  pound to 41,800 pounds.

This period of field work enabled the party to improve its operating technique and brought to light many minor difficulties in the mechanical operation of the equipment. Coupled with these difficulties was the fact that the many variable factors encountered in regular quarry blasting practice could not be controlled sufficiently for the effect of any one factor to be studied independently of the others; in addition, a quarry blast once shot could not be repeated under identical conditions. Moreover, as in previous tests, some records showed results that appeared to be contradictory. Experience gained in this field work indicated probable explanations for the erratic results, but because facilities were lacking for exact duplication, the results could not be checked.

For the foregoing reasons, conclusions drawn from interpretations of the records were offered as tentative only, pending further study and confirmation. Some of the tentative conclusions follow.

1. It was observed that for comparable distances from the shot and equal explosive charges the displacement measured with the seismometers on rock outcrops was generally about one-tenth that with the instruments on overburden. Application of this information explained the apparent contradiction in several of the tests in which larger displacements were recorded at greater distances. In all such

<sup>9</sup> The accelerations in the scale were inadvertently attributed to Wood and Neumann but should have been credited to Seiberg (28).

tests either the farther station was on overburden and the closer one on an outcrop, or both were on overburden whose physical characteristics differed in the two settings.

2. Where conditions were comparable between seismometer stations the trend in change of amplitude indicated a high degree of initial damping followed by a much lower degree as the distance from the shot increased. The initial damping was of the order of the inverse cube of the distance, whereas later damping was inversely proportional to the distance. Thus, the degree of displacement diminished as the distance from the shot increased, but the rate of decrease was variable.

3. The change in amplitude with change in weight of explosive shot, when the distance remained constant, tended to increase with increase in the explosive charge but at a rate somewhat slower than in direct proportion. Thus another variable factor was introduced.

4. Although the field work demonstrated conclusively that several variable factors were present that could not be evaluated, an attempt was made to develop a formula from which the displacement could be calculated if the weight of the explosive charge and the distance from the shot were known. This and other formulas will be discussed later. It is interesting to note, however, that the formula was used to check the amplitudes empirically determined in a number of field tests and that, whereas in a few tests the difference between observed and calculated amplitude was large, in most tests the ratio of observed to computed amplitude was 1:3 or better. As it is impracticable to evaluate the effect of stratigraphy, faulting, and similar geologic conditions, it was felt that the formula as developed was remarkably satisfactory for a beginning.

5. Computations were made to determine the acceleration of the vibrations in a number of tests for comparison with the Mercalli scale. Formulas for determining acceleration from frequency and amplitude are based upon the assumption that the vibration is sinusoidal. This is not true of seismic waves from quarry blasts, except in rare instances. Not only did the amplitude vary considerably while the vibration lasted, but also the frequency varied; however, some of the observed wave shapes approached sinusoidal, and as a matter of interest the computations were made. Curiously, the accelerations computed upon this basis compared with "serious" to "total" damage on the Mercalli scale.<sup>7</sup> Obviously, this reduces to an absurdity as no damage was done in any of the test shots. The authors therefore concluded that the Mercalli scale for determining the effect of vibrations from earthquakes cannot be applied to vibrations emanating from quarry blasts.

#### PRELIMINARY STUDIES OF BUILDING VIBRATIONS (42)

During 1936 the National Bureau of Standards conducted experiments to determine the severity of vibrations to which the walls and floors of buildings were subjected under normal living conditions and street traffic. The Bureau of Mines was requested to cooperate by using its seismometers to measure the displacement. This afforded the Bureau an opportunity to study the characteristics of house movement induced by a crude vibrator and passing street traffic.

<sup>7</sup> See footnote 6.

The tests were concerned with determining the maximum amplitudes of vibration possible in the floors and walls of a typical frame dwelling from everyday sources of vibration and from vibrations induced by a  $\frac{1}{2}$ -horsepower electric motor driving a small unbalanced flywheel.

The building was subjected to tests to determine its reaction as a unit and the reaction of various floors and walls vibrating as separate panels.

The tests were successful in determining the maximum motion induced by several types of agitation. For example, a 14,000-pound truck with solid-rubber tires driven over a 1-inch plank in the street 63 feet in front of the house caused a maximum movement on the second floor of 0.0069 inch. A second similar test gave a movement of 0.0062 inch. An 8,500-pound truck with pneumatic tires gave a maximum movement on the second floor of 0.0026 inch.

Maximum recorded movement of floor panels with a motor vibrator was 0.0019 inch.

Vibrations caused by slamming the front door on the first floor reached a maximum of 0.0088 inch. The greatest movement (0.01 inch) was caused by a 160-pound man jumping a few inches in the air and dropping back stiff-legged on the floor.

These tests were compared with earlier tests of quarry shots in which seismometer records were made in frame houses. The house movement from quarry blasting ranged from a minimum of 0.00015 inch, caused by the detonation of 1.13 pounds of explosive 715 feet from the house, to a maximum of 0.033 inch, caused by the detonation of 17,250 pounds of explosive 1,810 feet away. Incidentally, of 11 quarry tests, only 2 gave maximum house movements exceeding that caused by the 14,000-pound truck. Seven of the quarry tests, involving the detonation of 36 to 1,200 pounds of explosive at distances ranging from 715 to 2,500 feet from the blast, gave less movement than that caused by the 8,500-pound truck on pneumatic tires.

From these and similar records of passing trains and highway traffic it was concluded that customary quarry blasting caused vibrations comparable with those from ordinary traffic and movements of persons within residences under ordinary living conditions.

The tests for panel vibration and vibration of the structure as a unit were not adapted to specific interpretation. There was evidence, however, that the structure responded in a different manner when excited from exterior sources in that the frequency of vibration was nearly constant for all parts of the house. When it was agitated by an unbalanced motor, the frequencies covered a rather wide range and were not constant at any one point. This erratic result was thought to be caused largely by the transfer of impulses back and forth from panel to panel, as the motor and its unbalanced flywheel were too small to move the house as a whole.

Resonant frequencies of various panels were observed, but these could not be built up by the small motor.

#### SUMMARY OF 1936 QUARRY TESTS (43)

It was recognized that the amount of actual ground displacement that took place during the entire duration of vibration was of primary importance. Preliminary tests showed that the maximum displace-

ment might occur at the beginning of the vibration or later, therefore it was necessary to analyze the relative movement in all three components at identical instants over the entire duration. For example, the maximum resultant motion is the vector sum of the two horizontal components and the one vertical component as measured by the seismometers. If at a certain instant an abnormally high vertical component was coupled with low displacements on the two horizontal components, the resultant motion might be lower than a similar record at another instant when all three components were of average size.

The investigators realized that a study of this phase of the problem would also show the actual amount and direction of movement of the

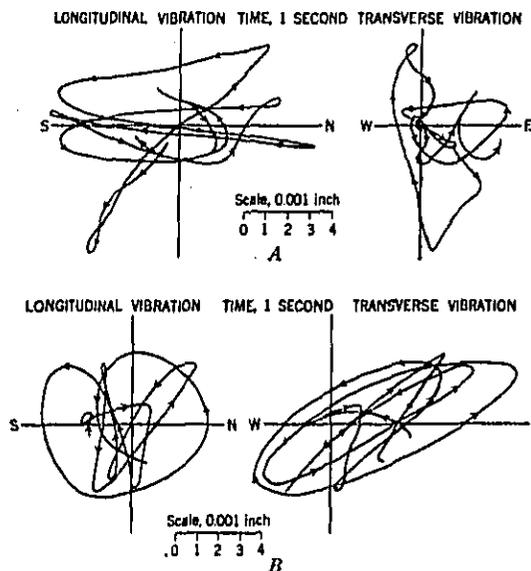


FIGURE 6.--Path of a vibrating point: A, Path of particle for 1 second resulting from a 13,400-pound shot 3,600 feet away; B, path of particle for the second following that in A.

particular point on which the seismometer station was set. They believed that knowledge of the path or direction of movement would help in subsequent interpretations.

#### PATH OF A VIBRATING POINT

It was found that if a single point (seismometer station) is observed, it vibrates in one direction *only* an *infinitely small fraction of time*; in other words, *the direction of vibration is constantly changing in three dimensions*. This fact is best-illustrated by figure 5, in which the motion of a single point is traced over 2 consecutive seconds. Two

diagrams are shown for each 1-second period, as the actual motion takes place in three dimensions and only two can be shown on a plane surface.

#### SPEED OF SEISMIC WAVE

The minimum speed of transmission of the seismic wave recorded in rock was 8,000 feet per second and the maximum was 28,300.

Table 1 gives the speed characteristics of various types of rocks as observed by the authors.

TABLE 1.—*Speed of seismic wave, feet per second*

	Minimum	Maximum
Biotite-gneiss.....	20,300	(1)
Dolomite.....	14,700	22,700
Flint.....	12,000	16,000
Gabbro-diorite.....	16,000	18,700
Limestone.....	8,000	18,000
Sandstone.....	11,000	(1)
Trap.....	15,500	17,000

<sup>1</sup> Only 1 measurement made.

#### AMPLITUDE OF SEISMIC WAVE

The smallest maximum resultant amplitude measured was 0.00006 inch, recorded at a station 7,440 feet from a shot of 100 pounds of explosive.

The maximum recorded amplitude was 0.058 inch, recorded 617 feet from the detonation of 15,400 pounds of explosive.

#### FREQUENCY OF SEISMIC WAVE

As the records of the seismic waves from quarry blasts showed irregular vibrations, perhaps it is misleading to refer to them as having a certain frequency; however, the term "frequency" as used in this connection refers to the predominant frequency observed on the record.

The frequency of the waves varied with the type of setting on which the seismometers were placed. On rock outcrops, frequencies ranged from 20 to 80 cycles per second. On overburden, frequencies ranged from 4 to 20 or more cycles per second.

In residential structures the observed frequencies were complicated. Although in some tests they coincided with those of the adjacent ground surface, in others no correlation was apparent. For example, at a house with one station outside on the surface of the ground and another inside on the second story, both stations vibrated at frequencies ranging from 20 to 22 cycles per second. At another house, the concrete basement floor and the ground outside both vibrated at 3.3 cycles per second. At a third house a station on the third story vibrated from 10 to 20 cycles, whereas one on the ground adjacent to the house vibrated from 16 to 45 cycles.

#### DURATION OF SEISMIC WAVE

The duration of the motion depended largely upon the characteristics of the terrain on which the seismometer station was placed. The minimum recorded was 0.1 second, on an outcrop 565 feet from a shot of 335 pounds of explosive. The maximum was 8.0 seconds on glacial fill 4,000 feet from a 13,400-pound shot.

## CORRELATION OF AMPLITUDE, FREQUENCY, AND DURATION

For equal explosive charges and distances from the shot, rock outcrops gave vibrations of lower amplitudes, higher frequencies, and shorter duration than overburden.

## CORRELATION OF AMPLITUDE AND DISTANCE

With other variables held constant, it was found that, in general, the amplitude of vibration decreases with an increase in distance from the shot.

*Correlation of amplitude with distance*

Rock	Explosive charge, pounds	Distance, feet	Station	Amplitude, inch
Blotie-gneiss	11,500	540	Outcrop	0.023
		1,100	do	.013
Trap	11,000	2,300	Overburden	.0016
		3,000	do	.0016
		3,500	do	.0017
Clabro-silicite	100	337	do	.0012
		2,130	do	.0004
		4,150	do	.00026
		7,410	do	.000033

## TESTS IN WHICH RESULTS WERE CONTRADICTIONARY

Rock	Explosive charge, pounds	Distance, feet	Station	Amplitude, inch
Limestone	213	1,470	Overburden	0.0007
		2,450	do	.0023
	163	1,470	do	.0007
		2,450	do	.0024
Trap	0,050	4,510	do	.0018
		5,000	do	.0033
		5,900	do	.0070

## TEST SHOWING NEARLY CONSTANT AMPLITUDE WITH DISTANCE DOUBLED

Rock	Explosive charge, pounds	Distance, feet	Station	Amplitude, inch
Limestone	543	520	Overburden	0.000
		1,000	do	.000

## CORRELATION OF AMPLITUDE AND WEIGHT OF EXPLOSIVE CHARGE

In most tests in which other factors have been constant the amplitude has increased with an increase in explosive charge.

*Correlation of amplitude and explosive weight*

Rock	Explosive, pounds	Distance, feet	Amplitude, inch
Dolomite	630	1,710	0.0015
	2,102	1,820	.0031
	3,256	1,710	.0043
	4,056	1,610	.0050

## EXCEPTION

Rock	Explosive, pounds	Distance, feet	Amplitude, inch
Flint	183	615	0.0018
	330	555	.0020

## EFFECT OF MOISTURE ON AMPLITUDE

No extensive tests were made to determine the effect of moisture; however, at one quarry two shots were recorded on consecutive days. The distances were approximately equal, but the weights of explosive charges differed. The seismometer stations were the same in both tests. The results follow:

*Effect of moisture on amplitude*

Rock	Station	Explosive, pounds	Distance, feet	Amplitude, inch
Limestone.....	Overburden.....	13,400	2,000	0.025
			2,540	.016
			3,030	.0082
Do.....	do.....	17,050	2,000	.021
			2,630	.014
			3,730	.008

The differences in distance were not considered sufficient to cause an appreciable change in amplitude. Although the second shot had 3,650 pounds more explosive, the amplitudes were slightly below those of the first shot.

The first day the ground was dry. The second day it rained continuously, and the ground was thoroughly soaked when the second shot was recorded.

This test indicates that the wet ground had greater damping effect on the seismic wave.

**EFFECT OF GEOGRAPHIC DIRECTION ON AMPLITUDE**

In several tests the vibrations from a shot showed greater amplitude in one direction than in another. This result was interpreted as reflecting differences in stratigraphic conditions.

**EFFECT OF KIND OF EXPLOSIVE ON AMPLITUDE**

In no quarry test was it possible to detect differences in amplitude due directly to the use of explosives of different strengths.

**EFFECT OF THICK OVERBURDEN ON FREQUENCY**

Where stations were set on overburden known to be thick (over 50 feet), the vibrations frequently showed high initial frequencies of short duration followed by prolonged low frequencies.

*Effect of overburden on frequency*

Rock	Explosive, pounds	Distance, feet	Overburden, feet	Frequency, cycles per sec.	Duration, seconds
Trap.....	19,500	1,350-2,330	98-112	30-60	1.0
				4-10	4.0
Dolomite.....	4,050	1,610-4,050	82-121	25-60	0.3-0.7
				5-10	6.0

However, in some instances the vibrations recorded by stations on thick overburden showed a characteristic low frequency of prolonged duration without any noticeable high frequency.

*Example*

Rock	Explosive, pounds	Distance, feet	Overburden, feet	Frequency, cycles per sec.	Duration, seconds
Trap.....	7,500	4,010	27	28	0.0
		4,510	90	6	5.0
		5,470	99	6	5.0

Figure 6 gives direct comparison of recorded amplitudes with sources of vibration, summing up the work of 1936.

SUMMARY OF 1936 QUARRY TESTS

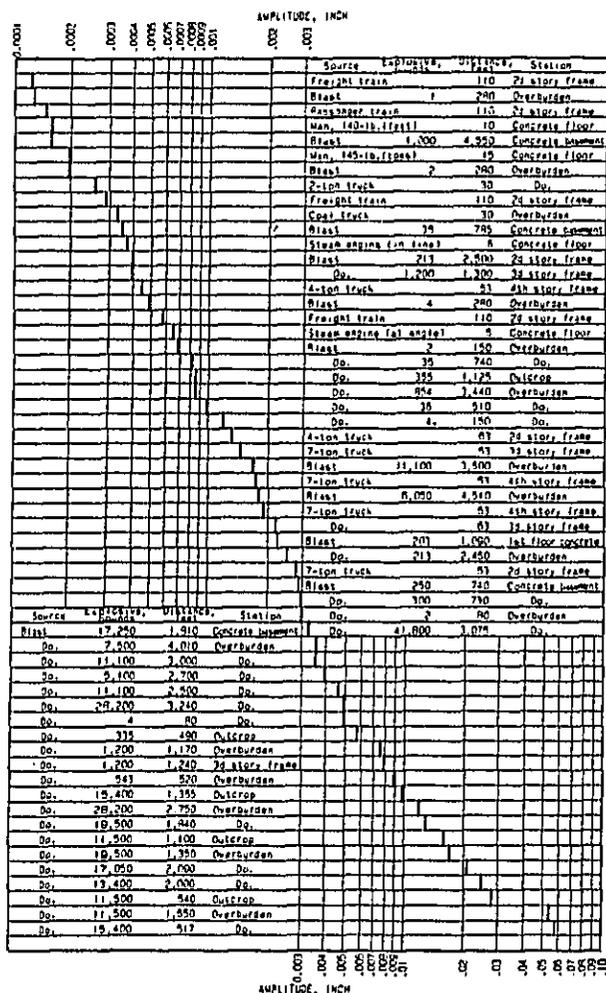


Figure 6.—Summary of recorded amplitudes, 1936.

## UNDERGROUND MINE TESTS (44)

Tests in open quarries brought to light many factors that influenced vibrations from quarry shooting. They also showed that the effects of these factors were variable and could be evaluated accurately only if each factor could be studied separately while other factors were held constant. For many reasons the customary practice in quarry blasting did not lend itself well to the rigid control necessary for accurate research. First, quarry shots customarily comprised simultaneous shooting of explosive charges placed in a number of large churn-drill holes. When fired, a shot usually provided the quarry with enough broken rock to last several weeks or even months, hence records had to be limited to a single shot fired under local conditions. These conditions were so variable that it was difficult to duplicate tests at any one quarry; moreover, shots at different quarries presented entirely different local conditions that were difficult to correlate. Moreover, even where conditions permitted and the operators were willing to suspend customary practice and prepare shots especially for test purposes, the variables could not be controlled within the desired range.

Means were therefore sought by which test shots could be made under rigidly controlled conditions and each variable factor isolated and studied independently. In the spring of 1937 an offer to conduct such tests was received from a company operating an underground limestone mine. The cooperation of an explosives manufacturer who wished to determine the vibrational effects of different grades of explosives was also obtained.

Arrangements were made to conduct comprehensive tests in which it was hoped to answer some of the questions left unanswered by tests in open-quarry blasting.

The three main objectives of these tests follow:

1. Determination of the natural frequency of vibration of the mine roof.
2. Determination of the character of vibrations owing to different quantities and types of explosives.
3. Determination of possible resonance between the natural period of the mine roof and vibrations produced by single- or delayed-shot blasting.

The answers to the first and third questions were desired by the mine operator to determine, if possible, whether vibrations produced by blasting were inimical to natural roof support.

The answer to the second question was desired by both the mine operator and the explosives manufacturer in order to devise a method of blasting that would be least harmful to the mine roof.

Other objectives, which had a more direct bearing on the research program in hand, included determination of the effect on the seismic wave of the following:

1. Use or absence of stemming in test holes.
2. Shape or concentration of explosive charge.
3. Position of test shot with relation to seismometer stations.
4. Shots that broke, compared with those that had too great a burden to break.
5. Shots delayed by intervals of a few thousandths of a second.
6. Changes in kind of explosive used.
7. Changes in amount of explosive charge.
8. Changes in distance of shot from recording points.

The first five tests were designed to ascertain within what limits field technique must be confined to obtain comparable results.

The last three were made to obtain data from which the effect of the principal known variables could be evaluated in future work.

#### INTERPRETATION OF TEST RESULTS

It is a well-known law of physics that a body composed of homogeneous material has natural periods of vibration. When such a body vibrates at any one of these periods the frequency will, of course, remain constant. If the body is subjected to an external vibrating influence of different frequency it will vibrate initially in a manner that is a combination of its own frequency and that of the agitating force. If the imposed vibrating influence is continued, ultimately the body will react at the frequency of the external force. If the external vibrating force can be arranged so as to impose a continuously increasing (or decreasing) frequency, the body will react with an increased amplitude when the frequency of the imposed force coincides with one of the natural frequencies of the body.

The last hypothesis is the basis of the claim advanced by several investigators that a house, having a natural frequency of its own, when vibrated by a seismic wave of the same frequency will vibrate in synchronism at an abnormally high amplitude. Under these conditions, only a few cycles of the seismic wave would be necessary to increase the vibration to dangerous, if not destructive, amplitudes.

It was thought that the stratum forming the mine roof might react to induced vibrations as a homogeneous body. If the natural frequency of the roof could be determined and an exterior force applied at that frequency, the roof might be made to vibrate in synchronism and the amplitude increased to the point of failure.

In all tests recorded in this mine the three seismometer stations were the same. The seismometers were set in specially prepared openings cut in the solid rock of the mine pillars.

#### NATURAL FREQUENCY OF MINE ROOF

Although numerous mechanical means were used to ascertain the natural frequency of the roof stratum, all of them failed to vibrate any large or extended area. The sum of all tests was interpreted to indicate that it was impossible to set the rock strata in vibration by mechanical means except over small, local areas. Such natural frequencies as were recorded for small, local areas were not comparable, hence it was concluded that the frequency of the roof stratum as a whole was indeterminate and that the recorded frequencies referred to local areas only. In other words, the roof stratum did not vibrate as a single unit over its entire area but vibrated as small local units in which the frequencies varied but did not differ greatly.

As the roof did not react as a unit there was little, if any, possibility of its vibrating in synchronism with an imposed vibrating force, such as might be caused by mine blasting, or of failure of a roof due to synchronous vibrations. This does not mean that loose pieces of the roof will not react to vibrations and fall. However, the tests indicate that the roof as a whole does not react dangerously to imposed vibrations.

## EFFECT OF STEMMING

Before discussing the results of these tests, the authors wish to make it clear that they were testing for the relative *vibrational* effect of tamped and untamped holes. This has nothing to do with the relative *disruptive* effect of tamped and untamped holes that are drilled to break rock.

The holes were prepared with equal charges of explosives, but one series of holes received no stemming whatever, a second series was tamped with two cartridges of stemming as in regular mine practice, and the third series was tamped solidly to the collars.

No difference was observed in vibrational effects from the detonation of tamped and untamped holes when the distance or weight of the explosive charge was held constant.

These tests indicated that vibrations from holes shot without stemming traveled in rock at an average speed of 11,400 feet per second, those from holes with two stemming cartridges at an average speed of 11,300 feet per second, and those from tightly tamped holes at an average speed of 11,150 feet. As the possible error in reading these velocities was approximately  $\pm 300$  feet per second, it was concluded that stemming had no effect on the speed of transmission of the seismic wave.

The records also showed that stemming had no measurable effect on the frequency or shape of the seismic wave.

## EFFECT OF CONCENTRATION OR SHAPE OF EXPLOSIVE CHARGE

It was considered essential to know what difference, if any, the shape of the explosive charge might cause in the vibration. For example, a single hole might be loaded with 10 pounds of explosive in cartridges laid end to end, so that the charge would be several feet long; the same quantity of explosive might be placed in a compact, spherical mass in the bottom of a chambered hole; or the charge might be divided and placed in two or more holes with several feet of rock between the holes. Such changes in the shape of the explosive charge might cause a significant change in the vibration for two reasons.

1. If a seismic wave begins at one end of an elongated charge and is followed by another from the other end the recorded wave shape might differ from that resulting from an explosive concentrated in a small sphere. It was felt that this test would apply particularly to the shooting of churn-drill holes in which the explosive train might be 50 feet or more in length.

2. The power exerted by an explosive is a function of time, therefore it was thought that an elongated charge detonated at one end might have less power than a concentrated charge because of the time required for the detonating wave to travel the length of the explosive train.

The results of six tests to determine the effect of shape of the explosive charge on the seismic wave indicated that the observed differences were within the range of possible error in reading the records. Hence, the charge could be detonated in the form of an elongated cylinder, in a compact mass, or in three separate cylinders separated by several feet of rock and the seismic effects would be the same, provided all parts of a separated charge were detonated simultaneously.

Likewise, no difference was observed in the frequency, wave shape, and speed of propagation of the wave owing to shape of the explosive charge.

#### LOCATION OF SHOT WITH RESPECT TO RECORDING STATIONS

The seismometer stations were in a straight line, with the No. 1 station nearest the area in which most of the test shots were made and in the same block of ground. The Nos. 2 and 3 stations, however, were in isolated pillars.

The question arose as to whether a shot made in the same block of ground as a recording station would give a record similar to that of a shot made in a block separated from the receiving station by mine openings. In the first instance, the seismic wave would travel direct from shot to seismometer through the intervening strata. In the second, the wave must travel up to and through the roof and down to the seismometer, down to and through the floor and up to the seismometer, or both.

Two shots of equal weights of explosive were made at the same point in a pillar isolated by mine openings from No. 1 station. Similar shots of equal weight were made an equal distance from No. 1 station but in the same block of ground.

The four shots showed no appreciable differences in amplitude, frequency, or speed of propagation. The wave shapes recorded from the two isolated shots were identical, as were those from the two shots in the same block, but the two pairs differed considerably in shape.

Although this difference in the shape of the wave record was interesting it had no great practical importance, as the amplitudes, frequencies, and speeds were the same; however, the problem of interpreting the differences in wave shape was assigned to Abraham Yanovsky, a member of the field party at that time. He made a special study of the shape of the waves recorded from all test points and was able to demonstrate conclusively that the position of a shot in the mine (within the experimental area) could be identified by the shape of the wave. This conclusion was important because it demonstrated the possibility of duplicating accurately shot conditions, even to the shape of the wave. Further study of the records showed that shots made for different purposes but under similar conditions produced seismic records that could be superimposed on each other with virtually no observable differences. The accuracy of the tests was thus satisfactorily demonstrated.

#### EFFECT OF SHOTS THAT BREAK ROCK COMPARED WITH THOSE THAT DO NOT

It was expected that the blast from a hole drilled in such a position that when loaded and detonated it would break rock, as is customary in mining practice, would produce less vibrational effect than a shot from a hole drilled straight into a wall with no chance to break rock. The reasoning was that as part of the energy of the explosive was used to disrupt the rock, only the remaining energy would be available for producing vibrations.

Several tests were made in which the quantity of charge, distance, and location of the shots were held constant, but one shot was drilled to break rock and the other so that it could not break; in addition,

different kinds of explosives were used in each pair of shots. In every test the break-out shots gave lower amplitudes of vibration, but the wave shape, frequency, and speed of propagation for each pair of shots did not differ materially.

These results were important, as they showed that for determination of fundamental vibration characteristics, test shots must be drilled so that they cannot break rock. Had the shots been designed to break rock as in customary mining practice, another variable would have been introduced, namely, the percentage of energy used in breaking the rock. As no two holes can be drilled in which identical breaking conditions can be assured, this variable could not have been accurately evaluated. All fundamental research tests therefore, were made with shots drilled so they could not break out.

#### EFFECT OF DELAYING MULTIPLE SHOTS A FRACTION OF A SECOND

The customary method of firing in this mine involved shooting several holes simultaneously as a single blast, followed by shooting a second and third group of holes similarly.

The test was designed to determine whether splitting the entire blast into three separate shots separated by only a fraction of a second would increase or reduce the vibrational effect.

A number of tests were made and compared, using shots of equal size without delay intervals. The resulting records gave no appreciable differences in amplitude, frequency, or velocity of propagation between delay and nondelay shots when the delay period was less than 0.015 second within the limiting conditions of the tests. Delays of more than 0.015 second resulted in misfires caused by flying rock from the first shot prematurely cutting the wires of the second.

Because of mechanical difficulties in the apparatus producing the delay in firing, this test was not considered sufficient to warrant the conclusion that all results with delay shots were equivalent to those with nondelay shots. The tests were instructive, however, in that no abnormally large differences were observed.

The shape of the wave recorded from delay shots differed materially from that recorded from nondelay shots; however, most of the wave shapes for the delay shots could be resolved into single waves characteristic of the shot location, each wave representing a single member of the delayed shot. The maximum amplitude of waves from delayed shots, however, was never greater than that from a single shot of corresponding weight; in other words, the vibrational effects from the several parts of a delayed shot did not unite to build up the amplitude.

#### EFFECT OF DIFFERENCES IN KIND OF EXPLOSIVE

Ten explosives were furnished to determine whether any difference in vibrational effect occurs with a change in kind of explosive. All the explosives were sold regularly on the market as commercial grades. Although they differed materially in physical characteristics, the differences in any one characteristic or its relation to others were insufficient to warrant a definite conclusion that differences in recorded results were due exclusively to one physical characteristic.

There was some slight evidence that straight-nitroglycerin-type explosives gave less vibrational effect than other commercial types.

Some evidence was shown that higher-grade (rated-strength) explosives gave increased vibrational effect, but the increase was too small to warrant definite conclusions.

A number of tests indicated that the vibrational effect increased with an increase in the rate of detonation of the explosive, but the relationship could not be evaluated.

Frequency, speed of propagation, and wave shape were unaffected by a change in kind or strength of explosive.

It was difficult to draw conclusions from these tests, owing to inability to isolate one characteristic of the explosive and test it with reference to the others. For example, the physical differences recognized were density, ingredient weight-strength, gas volume, and rate of detonation. Density and rate of detonation are relatively easy to determine by physical tests; ingredient weight-strength and gas volume, however, are subject to different interpretations. As none of these characteristics was determined in Bureau laboratories, their exact definition is impossible.

Furthermore, the explosives employed in the tests were of commercial grade and as such did not have any one physical quality in excess.

Although inconclusive, the tests indicated that explosives that combine high weight-strength and gas volume with low density gave higher vibrational effect when the rates of detonation were equal.<sup>5</sup>

The authors concluded that differences in the physical characteristics of the explosives were of minor importance and were greatly overshadowed by difference in weight of explosive charge, regardless of the kind of explosive used.

**EFFECT OF WEIGHT OF EXPLOSIVE CHARGE AND DISTANCE BETWEEN SHOT POINT AND SEISMOMETER**

In this series, each test was made with all known variables held constant except the one under study. For example, when testing for the effect of change in weight of explosive charge, the shots were made at equal distances from the seismometers, and all holes were loaded in the same manner with explosives of the same kind, the only difference being the weight of the explosive fired. Likewise, in testing for the effect of distance, equal weights of the same explosive were loaded in the same manner, the only difference being the distance between the point of firing and the seismometers.

<sup>5</sup> Subsequently, a series of tests was made at the Bureau's Experimental adit at Mount Weather, Va., in which four explosives furnished by a manufacturer were tested. The explosives were not commercial grades but were special mixtures made for the test in which "brissance," gas volume, and rate of detonation were the three physical characteristics emphasized. These were combined in the four mixtures as follows:

Explosive mixture	Brissance	Gas volume	Rate of detonation
A.....	Low.....	Low.....	Low.
B.....	do.....	High.....	Do.
C.....	High.....	Low.....	High.
D.....	do.....	High.....	Do.

The tests indicated that gas volume predominated in producing vibrational effect. The explosive mixtures were not tested by the Bureau for their physical characteristics, and the relative values were as reported by the manufacturer without exact definition. Hence, the tests indicate a possibility that certain physical characteristics may produce greater seismic effect than others.

Tests were run with each of 10 kinds of explosive, but the weight of charge with one kind was not necessarily the same as that with another, nor were distances equal for all kinds. For each explosive, however, the weight or distance was the only item changed.

In all tests the amplitude of vibration increased with an increase in the weight of the explosive charge. Moreover, for a given weight of charge, the amplitude decreased with an increase in the distance between the point of firing the shot and the seismometer. The decrease in amplitude with an increase in distance is very rapid at short distances but comparatively slow at greater distances. The distance at which this change occurs appears to be a function of the weight of the explosive charge. With large charges, the change occurs farther from the shot. In the tests made, the change from rapid to slow damping occurred between 200 and 400 feet from the shot.

Before this series of mine tests was concluded another attempt was made to evaluate the relationship of amplitude to the weight of explosive charge and distance. This relationship is discussed in the section on mathematical formulas.

#### **BLASTING VIBRATIONS CARRIED TO STRUCTURAL DAMAGE (45)**

Up to the time the tests were completed in the underground mine, the primary objective had been the collection of fundamental information relative to the characteristics of seismic vibrations. With the exception of the tests made in collaboration with the National Bureau of Standards and a few records made in available residences in conjunction with the quarry tests, no attempt had been made to study the effect of the seismic disturbance on residential structures.

With the completion of the mine tests, however, it was felt that enough fundamental knowledge had been obtained to warrant extending the study of seismic waves to their effect on buildings. Accordingly, an attempt was made to locate buildings that could be used for experimental purposes. It was realized that this might be difficult because of the rigid requirements under which the tests would have to be conducted. In the first place it was doubtful if tests carried to the damage point would be permitted in any structure unless it had already been scheduled for demolition. It was realized also that any building scheduled for demolition probably would be in such a state of disrepair as to be useless for test. Suitable structures within city limits frequently are torn down to make way for more modern construction while they are still usable, but here the danger was that adjoining buildings also might be damaged by the force of the blasts necessary to damage the building under test. Moreover, city ordinances usually prohibited the use of explosives in the quantities thought necessary. The problem was complicated by the difficulty of placing the blasts so that the seismic wave alone would affect the structure. From incidental tests already made, the air wave from a blast was known to cause considerable movement of a building. To insure that the resulting damage was the result of the seismic wave alone, it would be necessary to shield the building from the air wave, which in itself would be difficult.

Several organizations were found that were willing to cooperate, but upon investigation each suggested location except one presented some unsurmountable difficulty.

At the Bureau's Experimental mine (32), Mount Weather, Va., the testing adit had been driven in solid rock to a point almost directly beneath a residence used as the mine office but originally built as a farmhouse. Minor remodeling had been planned to make the building more suitable for an office, and any damage resulting from tests could be repaired during remodeling. This location had the further advantage that blasting could be done in the adit beneath the building

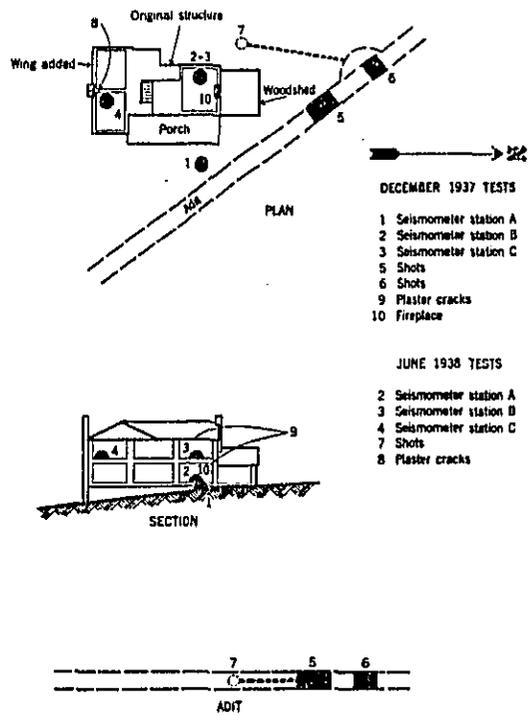


FIGURE 7.—Experimental adit and office.

without any possibility of an air wave confusing results, as the portal of the adit was 400 feet from the house.

The original plaster of the house was badly cracked and had fallen in some places. It was decided to run one series of tests as the building stood and a second series some months later. Meanwhile, one room was replastered, and the second series of tests was delayed until the new plaster had ample time to assume a permanent set.

In the first series of tests, begun in December 1937, the blasts were set off in the adit 70 feet vertically below the foundation of the house and 30 feet north of the center of the building. (See fig. 7.)

Three seismometer stations were used—one on the surface of the ground just outside the house, one on the first floor, and one on the second floor.

Charges ranging from 10 to 195 pounds were fired in holes drilled in the adit heading.

No damage was observed until the 195-pound shot was fired. This shot opened a crack in the plaster of the first-floor ceiling parallel to and about 6 inches from the nearest wall directly above the shot. The wall contained a brick fireplace and was strengthened by the chimney extending through it.

A second minor crack appeared in the second-floor ceiling, but it was only about 14 inches long and extended between two old cracks.

The amplitude measured on the first floor was 0.09 inch and on the second floor 0.08 inch.

From careful inspection during and following the tests, the authors interpret the damage represented by the crack parallel to the wall as having been caused by relative movement of the parts of the house rather than by panel vibration of the first-floor ceiling. Six months later (June 1938) similar tests cracked new, well-set plaster.

In this series the blasts were set off in a long hole driven in the wall of the adit so that the explosive charges were 65 feet below the foundation and 10 feet northwest of the house. Four shots were made which ranged in size from 5 to 150 pounds.

With a 75-pound explosive charge the seismometers recorded a house movement of 0.1 inch, and no damage was observed. With the 150-pound shot the movement extended beyond the measurable range of the seismometers but was estimated to have been between 0.2 and 0.3 inch.

As no cracks appeared with the 75-pound shot, it was evident that the new plaster successfully withstood a movement of one-tenth inch. No damage was observed to old plaster in other parts of the house in any tests of the second series, and no old or new ceilings failed.

The 150-pound shot, however, cracked new plaster in two places adjacent to a chimney in an outer wall. Careful examination of the construction of this chimney revealed that it was free to move independently of the wall of the house but was tied to the plaster.

The authors' interpretation, as in tests of the first series, was that the plaster failed owing to relative movements of parts of the house and not to panel vibration nor vibration of the house as a unit.

After these tests at Mount Weather, all previous tests in which records had been made in houses during quarry blasting were reviewed for comparison. (See table 2.) Of the 13 records so obtained, the highest amplitude recorded was only 22 percent of that required to break the plaster at Mount Weather, and the average of the 13 was not more than 5 percent.

HOUSE MOVEMENT INDUCED BY AGITATION AND BLASTING 23

TABLE 2.—Building vibrations from quarry shots

Explosive, pounds	Position of recording station			Displace- ment, inch
	Floor	Type of construction	Distance from disturbance, feet	
17,250	Basement.....	Concrete.....	1,810	0.033
1,200	Third.....	Wood.....	1,240	.004
8	Second.....	Stone.....	233	.004
300	Basement.....	Concrete.....	730	.003
250	do.....	do.....	740	.0029
203	First.....	do.....	1,000	.00225
8	do.....	Stone.....	228	.002
213	Second.....	Wood.....	2,000	.0004
163	do.....	do.....	2,300	.0004
1,200	Third.....	do.....	1,300	.0004
30	Basement.....	Concrete.....	785	.00035
1,000	First.....	do.....	4,550	.00016
1.13	Basement.....	do.....	715	.00016

HOUSE MOVEMENT INDUCED BY MECHANICAL AGITATION AND QUARRY BLASTING (47)

The tests at Mount Weather indicated that a far greater movement was necessary to cause damage, as measured by failure of plaster in a typical residence, than that resulting from customary quarry blasting at distances usually found between quarry shots and buildings. Moreover, as the tests were confined to one building, they could not be applied to other localities where conditions might differ. Additional tests at Mount Weather were useless for the same reason. Again, an attempt was made to find suitable locations for continuing similar tests, but after diligent search none was discovered.

It was therefore decided that other means must be used to compare the response of various structures to seismic waves.

The published records of other investigators (2, 19, 20) on the effect of mechanically induced vibrations at earthquake intensity on built-up panels and miniature structures indicated the difficulty of correlating laboratory-scale tests with actual conditions in the field. This is evident from the fact that all buildings are composed of floor and wall panels tied together more or less rigidly and that each panel has its own natural period of vibration and is capable of communicating its movement to the four or more connecting panels. Any change in the dimensions of a panel or in the materials from which it is constructed may change the natural vibration response of that panel. Hence, miniature structures or test panels are not satisfactory for indicating susceptibility to damage of full-scale buildings of the same materials when subjected to outside sources of vibration.

It was recognized that, in order to be conclusive, tests must be made with full-scale buildings and not with miniatures or panels.

In addition to these difficulties, the study had shown conclusively that vibrations from quarry blasts were not always consistent in amplitude, frequency, or duration, even when contributing factors were closely controlled. This does not mean that tests cannot be repeated with identical results, but it does mean that any vibration emanating from a blast consists of a series of complicated waves in which the amplitude and frequency change continually. The series

as a whole may repeat itself exactly for two shots in the same locality fired under identical conditions.

As it was impossible to find suitable locations for further tests of actual blasting, the only alternative was to test full-scale buildings with mechanically induced vibrations, then to compare the results with quarry tests.

The problem was presented to quarry operators and their assistance requested. Response was immediate, and numerous buildings were offered for test. This cooperation was particularly gratifying because no restrictions were placed by the operators on the extent to which the test structures could be damaged. In fact, perfectly sound residential structures were offered for test, and the authors were told to demolish them completely if necessary to obtain the desired results.

The next step in the program was to find a suitable means of mechanically vibrating the structures. The vibrating mechanism must be so designed that it could produce impulses in a single direction; the speed or frequency must cover a wide range and be controlled so that any desired speed could be held within a very narrow range; furthermore, the design must be such that the magnitude of the force developed by the vibrator could be varied at will over a rather wide range. In addition the vibrator must be capable of delivering either simple horizontal or vertical impulses.

Several types of commercial vibrators were investigated but for one reason or another failed to meet these rigid requirements. The United States Coast and Geodetic Survey (52, ch. 7) had conducted similar experiments with a mechanical vibrator, but this machine was designed to produce impulses in the range of frequencies encountered in earthquake phenomena and was unsuited to producing the higher frequencies of quarry blasts. Attempts were made to design a vibrator for this work, but bids received for its construction were beyond the limit of available funds. Finally, it was found that the Baldwin-Southwark Corporation, Philadelphia, Pa., had an experimental vibrator that seemed to fit requirements. This company agreed to lease the machine for sufficient time for a comprehensive series of tests.

Work was begun on tests using this vibrator in the spring of 1939 and continued throughout the summer.

A number of houses close to active quarry operations were selected so that records of customarily large quarry shots could be made for direct comparison with shaker tests.

Of the buildings chosen for initial tests, two were frame, one was brick, and one was stone masonry. Later, other frame buildings were tested, both for reaction to the vibrator and to quarry blasts under different stratigraphic conditions, to confirm the results obtained in the initial structures.

The recording equipment, comprising the seismometers, oscillators, and oscillograph, was the same as that employed in previous tests.

The vibrator or shaker used to produce mechanical vibrations was an "unbalance rotor" type driven by an electric motor. This machine consists of two wheels geared together to revolve in opposite directions at the same speed. (See fig. 8.) Each wheel carries one fixed weight and one adjustable weight (free to move along the circumference). If the adjustable weights are placed diametrically opposite the fixed weights the unbalance is zero, therefore the centrifugal force is also

zero. If the adjustable weights are moved adjacent to the fixed weights the degree of unbalance is maximum.

Because the machine has two wheels that revolve in opposite directions horizontal forces are canceled, and only vertical force is produced when the machine is upright. For example, a study of figure 8 shows that when the force of the left wheel is directed to the right the right wheel produces a force of the same magnitude to the left; hence, the

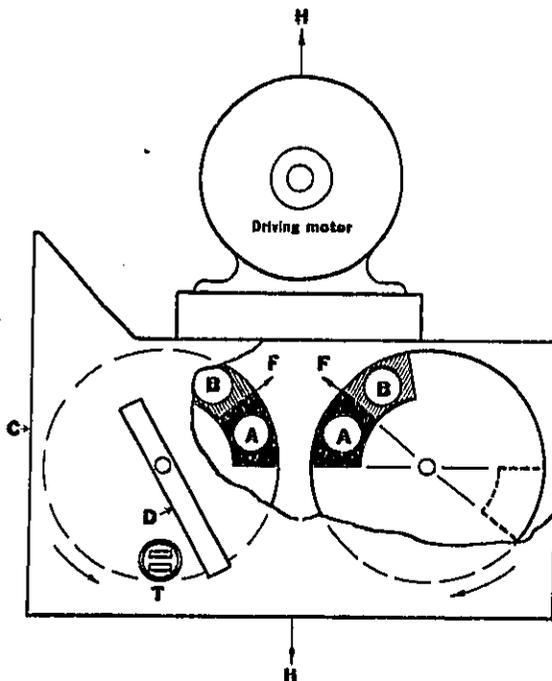


FIGURE 8.—Mechanical shaker: A, fixed weight; B, movable weight; C, base for horizontal force; D, speed-phase bar; F, force; H-H, line of force; T, telephone earpiece.

forces cancel and both the horizontal and vertical resultants are zero. When the force of the left wheel is directed to the right and up, the force of the right wheel will be to the left and up. The horizontal components of the forces of the two wheels are equal but opposite, hence they cancel each other, but the vertical components are equal and in the same direction and thus can be added. When the force of one wheel is directly up (or down) that of the other is also up (or down), and the total force is twice the force of one wheel. There is no horizontal force on either wheel when the force is directly up (or down); hence, the resultant horizontal force is still zero. The net re-

sult is a simple vertical force that varies sinusoidally with time. The frequency of the force depends on the speed of rotation, which, in turn, can be controlled by regulating the speed of the driving motor.

In order to produce pure horizontal force, the vibrator is turned 90° from the upright position.

The machine is characteristic of centrifugal devices in that the force varies as the square of the frequency. For a given frequency, the force delivered depends on the degree of unbalance, which is regulated by the position of the movable weights.

The machine is limited by its mechanical construction to a maximum force of 1,000 pounds. The maximum speed is 2,500 r. p. m., or about 40 cycles per second.

For complete analysis of tests it was necessary to know the phase angle between the force and the displacement. Accordingly, a phase-indicating device was mounted on the vibrator which, through suitable electrical connections, produced an identifying mark on the record once every revolution of the wheels. This mark on the record corresponded to a known position of the unbalanced weights. With this position and the speed of the vibrator known, the magnitude and direction of the force could be determined for every instant on the record. Comparison of the force marks and displacement curves on the record gave the phase difference between the two. The general plan of procedure follows:

1. Determine the vibrational characteristics of the house as a whole.
2. Record the vibrations from quarry shots.
3. Determine the characteristics of the floor (or wall) panels.
4. Vibrate the building or panel to the point of damage to obtain data on the index of damage.

In all tests of quarry shots three seismometer stations were set up in the house (on three floors), and a fourth was placed on the surface of the ground adjacent to the house so as to correlate ground and building vibrations.

In determining the vibrational characteristics of the building as a whole the vibrator was tightly blocked in a doorway and oriented to produce horizontal motion. Two settings at right angles to each other were always made with horizontal forces.

In testing floor panels for vibrational characteristics the shaker was fastened to the center of the floor in position for vertical force.

In testing for natural frequencies in either the house as a whole or a panel, the shaker was taken to full speed, the current was cut off, and the wheels were allowed to coast to rest. The centrifugal force was set low enough so that damage would not occur but large enough so that satisfactory seismometer records could be obtained, particularly for determination of the natural frequency or frequencies of the building or panel.

For damage tests the vibrator was set at a predetermined unbalance, the motor was started, and the speed was raised to the desired point. When the desired frequency, for example a predetermined natural frequency, was reached, the speed was held constant for 2 minutes or longer.

As the ability to maintain a particular speed (frequency) within narrow limits was of paramount importance, the vibrator was calibrated against the time-recording device in the oscillograph at various speeds, and satisfactory control was demonstrated.

## INTERPRETATION OF TESTS

In all, 14 structures were tested to determine vibrational characteristics, damage indexes, or comparative effect of quarry blasting. The buildings were frame, brick, or stone masonry and ranged from one to three stories in height with a width or length about equal to their height. No relatively tall buildings were tested, nor were rigid types of construction tested, such as reinforced-concrete and steel buildings.

The three main objectives of these tests were:

1. Determination of the characteristics of the modes or manners in which structures of residential type vibrated.
2. Correlation of the effects of mechanically induced vibrations with seismic vibrations from quarry blasts.
3. Ascertainment of an index of damage for seismic vibrations.

In addition, answers were sought to several other questions as the study progressed.

## DAMAGE AT RESONANCE

One of the secondary objectives of the tests was confirmation or refutation of a hypothesis frequently advanced that abnormally high or destructive displacements could be produced by driving a house at its resonant frequency. The theory advanced was that even a very small external force, if in resonance with the natural frequency of the house, would build up the amplitude of vibration to dangerous proportions in a short time. The theory also embraced the hypothesis that a large force (not destructive in itself) acting in resonance with the house would build up the amplitude to dangerous limits in only a few (two or three) cycles. Thus, according to the theory, a seismic wave from a quarry blast, even though it lasted only a fraction of a second, carried potential damaging ability if in resonance with the structure.

A study of the fundamental characteristics of seismic waves from quarry blasts had shown that in the majority of records the duration of the disturbance was of the order of 1 second or less. Where durations were longer, the cause was usually the physical condition of the ground beneath the building. It was also observed that in disturbances of short duration neither the frequency nor amplitude of the vibrations was constant and the direction of motion changed very rapidly. Hence, there seemed to be little likelihood that such a wave, with its constantly changing characteristics, would excite a building at resonance. Where longer durations were observed, the wave often approached sinusoidal conditions at low frequency and would be more apt to vibrate in resonance.

Attempts to excite a mine roof had shown that the roof stratum did not react as a unit. The question remained, however, whether a house or single panel within the house could be made to react as a unit.

The method used to detect the resonant frequencies of a house or panel was to record the motion during the coast-down test with the shaker. As the reducing speed of the shaker passed through the natural frequencies of the building or panel, resonance was indicated by increased amplitude on the record. The increased amplitude resulted from building up the vibration by resonance between the shaker and the building. As the shaker gradually slowed down, however, the length of time its speed would approximate the natural

frequency of the building would be short, and the question arose as to whether the resonant peaks produced in the short time (at resonance) of a coast-down test were maxima. In other words, if the shaker were set to revolve at a speed coincident with the natural frequency of the building, would the resonant amplitudes continue to build up with time?

A typical test, confined to panels, was made on two ceilings in the same building, and the results follow. In ceiling A, the coast-down test showed the natural frequency of the ceiling panel to be 12.2 cycles a second and the maximum resonant amplitude 0.023 inch. With no change in the position of the unbalance weights on the shaker (keeping the force constant with that used in the coast-down test) but with the shaker run at 12.2 revolutions per second, the amplitude never increased above the 0.023 inch recorded in the coast-down test. This test was repeated in all houses with the same result. It was evident then that driving the panel at resonance with the shaker did not increase the amplitude above that reached at resonance in the coast-down tests. In other words, the maximum resonant amplitude could be attained in the short time the shaker was close to resonance when coasting.

In another building this test was continued for 1 hour with the same results.

The shaker was then adjusted for greater force, and ceiling A was driven again at resonant frequency (12.2 cycles per second). The amplitude increased to 0.4 inch, indicating an increase in movement due to greater force at resonance, but even at this maximum the plaster was not damaged. The plaster was damaged, however, at 17 cycles per second, with an amplitude of 0.20 inch. In other words, damage occurred at a lower amplitude than that reached at resonance but at a higher frequency.

In ceiling B the natural frequency was determined at 12.4 cycles per second, and the maximum amplitude attained in both coast-down and driving-at-resonance tests was 0.06 inch. This ceiling was carried to a maximum amplitude of 0.3 inch and a frequency of 33 cycles per second with various combinations of force on the shaker without damage to the plaster. In fact, it was impossible to shake the ceiling within the limits of force available (1,000 pounds) to an extent that would damage the plaster.

In tests of a third ceiling, C, in the same building, however, the plaster fell at an amplitude of 0.15 inch and a frequency of 11.6 cycles per second (resonance) during the coast-down test. Before the test it was observed that a large area of plaster on the ceiling was hanging loose from the lath. This was the only panel tested in which damage was caused at resonance.

With the exception of ceiling B, all ceilings in the several houses were vibrated to the damage point, but damage in each instance occurred at frequencies having no relation to resonant conditions.

These tests showed that the physical condition of the plaster was of primary importance; in other words, plaster could fall if it was already weakened or loose. On the other hand, they also indicated that even with a force of 1,000 pounds acting at various frequencies the plaster could not be broken if it was in exceptionally good condition.

In all other tests on plaster of average physical condition, the amplitude could not be increased abnormally by driving at resonance, and when damage did occur it was at some other frequency.

The tests also showed that amplitude at resonance was to some extent a function of the applied force, because as the force increased the amplitude increased, but the amplitude at all other frequencies also increased.

Except where the plaster had been weakened previously, these tests apparently refute the theory that an incoming vibration, if in resonance with the natural frequency of a panel, could build up displacement to dangerous proportions within a few cycles.

Although C was the only ceiling damaged at resonance, this fact does not mean that damage may not occur at resonance, provided enough force is applied. However, a force large enough to cause damage at resonance will also produce displacement in the damaging range at other frequencies. In other words, resonant frequency is not necessary to cause damage.

Similar tests were made in which the entire structure, as distinguished from a single panel, was driven at the predetermined resonant frequency of the structure. In all such tests, driving at resonance without changing the shaker force did not produce greater amplitudes of motion than were obtained by the coast-down method.

Up to this point, tests of the house as a whole paralleled those on single panels. However, it proved impossible to drive an entire structure at resonance or at any other frequency to an extent where damage occurred. This was probably due to the fact that the maximum force of 1,000 pounds of which the shaker was capable was inadequate. Nevertheless, in every test it was possible to drive the house (without damage) to amplitudes considerably above those recorded from nearby quarry shots of the size employed in regular quarry practice.

It is important to remember that the amount of sway of a building is affected by the incoming vibration, especially near resonance. Two quarry shots of different frequencies that produce approximately the same displacement at the ground station near the house may produce differences in the amount of movement at the top floor of as much as 100 percent. In other words, where the disturbing force is unlimited in power (as it might be from a large quarry shot very close to the house) the resulting amplitude of the house movement will be increased if the incoming vibration is at or near resonance. The increase, however, will not reach the damaging range because of the damping characteristics inherent in the house itself. This statement must be considered as applying to quarry shots within the range of magnitude and distances covered in the tests. Obviously a house would be demolished if it were only a few feet from a large quarry shot.

It should also be remembered that driving a building or panel at resonance with the shaker produces greater displacement than vibration from a seismic wave, because the motion produced by the shaker is reciprocal in a single plane and sinusoidal and the panel or building responds in a similar manner, whereas a seismic vibration, except under specific conditions, is not sinusoidal but moves irregularly, changing direction in three dimensions with great rapidity.

## EVALUATION OF DAMPING CHARACTERISTICS OF A BUILDING

A second objective of these tests was to develop a means of evaluating the damping factor of a building or its resistance to vibration.

It is a well-known law of physics that when a simple mechanical system (single degree of freedom, simple harmonic motion, and viscous damping) vibrates at its resonant frequency, the amplitude of displacement at resonance is limited only by the damping factor or ratio of resistance (frictional forces) to twice the mass. The damping factor is therefore a measure of the susceptibility of a structure to vibration at its resonant frequency, provided the structure reacts as a simple mechanical system.

The ratio of the damping factor to the natural frequency is called the decrement. The decrement is a measure of "sharpness" of resonance—a low decrement gives a sharp peak and a large decrement a broad, flat peak—also of decay of amplitude in a free vibration.

The damping factor (hence the decrement when the natural frequency is known) can be determined by different methods. The two methods employed in these tests were determination of damping (1) from the variation of the phase angle between force and displacement as the frequency passed through the resonant value and (2) from the sharpness of the amplitude-frequency curve.

The validity of either procedure depends, of course, on the assumption that residential structures react as simple mechanical systems. One method of showing the similarity between the ideal (as represented by a simple system) and actual (as represented by a house) cases is to construct a displacement-frequency curve from the theoretical expressions for the ideal assumption and to compare the curve so constructed with the observed displacement-frequency curve obtained in the field.

The amplitude of vibration for an ideal case can be expressed as shown by Wood (55, p. 37):

$$a = \frac{f \sin \epsilon}{2kp}$$

in which  $a$  = maximum amplitude of vibration,

$f$  = force per unit mass ( $F/m$ ),

$\epsilon$  = phase angle,

$k$  = damping coefficient, and

$p$  = forced angular velocity.

With a force such as that developed by the shaker,

$$f = \frac{F}{m} = \frac{m_1 r_1 \omega^2}{m}$$

where  $m_1$  = unbalanced mass of shaker,

$r_1$  = radius of unbalanced mass,

$m$  = mass of vibrated body.

Substituting this value for  $f$ ,

$$a = \frac{m_1 r_1}{m} \times \frac{p \sin \epsilon}{2k}$$

but as shown by Wood,

$$\tan \epsilon = \frac{2kp}{(n^2 - p^2)}$$

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where  $n$ =natural or free angular velocity. Thus,

$$\sin \epsilon = \frac{2kp}{\sqrt{(n^2-p^2)^2 + 4k^2p^2}}$$

Substituting this value for  $\sin \epsilon$ ,

$$a = \frac{m_1 r_1}{m} \frac{p^2}{\sqrt{(n^2-p^2)^2 + 4k^2p^2}} \quad (1)$$

From equation 1 it can be seen that zero damping ( $k=0$ ) will result in infinite amplitude at resonance (where  $n=p$ ). For some finite value of damping the variation of amplitude with frequency can be determined from the equation; that is, as  $p$  is varied  $a$  will have corresponding values.

Figure 9 gives the displacement-frequency record for one of the houses tested. From the slope of the phase curve (not shown), the damping factor was calculated to be  $3 \pm$ . The natural frequency is

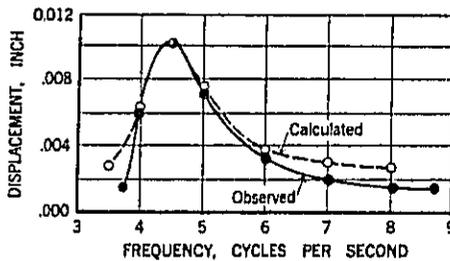


FIGURE 9.—Observed and calculated displacement-frequency curves.

approximately the frequency at which maximum amplitude (0.01 inch) occurs, or 4.5 cycles per second.

Substituting these values of damping and natural frequency in equation 1 for the resonant conditions (where  $n=p$ ) gives,

$$0.01 = \frac{m_1 r_1}{m} \frac{[2\pi(4.5)]^2}{\sqrt{4 \times [3.20 \times 2\pi(4.5)]^2}}$$

or

$$0.01 = \frac{m_1 r_1}{m} \frac{\pi(4.5)}{3.20}$$

$$\frac{m_1 r_1}{m} = \frac{0.01 \times 3.20}{4.5\pi} = 0.0023.$$

Equation 1, with the constant  $\frac{m_1 r_1}{m}$  evaluated, is now written

$$a = 0.0023 \frac{p^2}{\sqrt{[(2\pi \cdot 4.5)^2 - p^2]^2 + 4(3.20)^2 p^2}} \quad (2)$$

and expresses the variation in amplitude with frequency provided the house responds as a simple mechanical system. Points calculated from equation 2 are plotted on figure 9 and result in the dotted curve. The similarity between the observed and calculated data is apparent.

From the foregoing reasoning and empirical results, the authors concluded that it was possible to compute a significant damping factor for any building from the results of properly conducted tests.

#### DETERMINATION OF MODES OF VIBRATION OF A BUILDING

The first main objective of these tests was to determine how a building vibrates. In a typical test the north-south natural frequency (along one of the major horizontal axes) of a building was determined at 4.4 cycles per second. At right angles (along the other horizontal axis) the natural frequency of the same building was 5.5 cycles per second.

A study of the movement of all three floors at these resonant frequencies showed that the amplitude of displacement increased from ground to attic. In addition, the vibrations on all three floors were in phase. This means that the house was swaying in the plane of one of its major axes or in "translational" motion. Furthermore, it was observed that the house was twisting about its vertical axis or in "torsion" and that, since the ground acts as a clamp, the angle of twist or "angular displacement" increased progressively from ground floor to attic.

Although it was known from the studies of others (10, 19, 52, 53) that tall buildings reacted in this manner, this is believed to have been the first reported record of buildings that were approximately equidimensional moving in such a manner.

#### CORRELATION BETWEEN EFFECTS OF MECHANICALLY INDUCED VIBRATIONS AND SEISMIC VIBRATIONS FROM QUARRY SHOTS

A study of the tests in this series showed many instances in which the position of the seismometers was the same when records were taken of vibrations produced by quarry shots and by the mechanical vibrator. In these directly comparable tests, when the order of magnitude of the shot vibrations and the shaker vibrations was the same, no damage resulted. To produce damage it was necessary to increase the severity of the shaker vibration far beyond the range represented by the quarry shots.

When the shaker was operated to vibrate the buildings as units, the form of distortion was very similar to that produced when the building was swayed by the seismic wave from a quarry shot.

Experimental blasting tests carried to the damage point at Mount Weather, Va., agree in the order of severity of vibration with shaker tests that produced damage.

In the foregoing respects the shaker tests agree closely with the quarry-shot tests. The significant differences are duration of agitation and wave shape.

Quarry-shot vibrations, except under special stratigraphic conditions, are irregular and persist only a second or fraction of a second, whereas shaker impulses are sinusoidal and continuous. As has been shown, however, their effects on buildings, as determined experimentally, are similar.

## INDEX OF DAMAGE FOR SEISMIC VIBRATIONS FROM QUARRY BLASTS

As cracks in the plaster of a house are associated (with or without justification) with damage from blasting far more than any other effect, it is convenient to utilize failure of the plaster in determining the index of damage.

It has already been pointed out that failure of plaster is a function of the physical condition of the plaster, therefore, extremes are ruled out and failure of plaster, as referred to in this paper, means plaster in average physical condition.

Even when this definition is taken as the measure of damage, failure of plaster is not a sharply defined event but rather a gradual transition. Visually, the initial indication of damage is the extension of old cracks or dust falling from the sides of the cracks as they are rubbed together. As the severity of the vibration is increased, fine new cracks are formed, and the plaster may flake or spall slightly or the surface or putty coat separate from the brown coat beneath. A further increase in severity of vibration causes extension of the new cracks and finally causes large areas to separate from the lath and fall. In this report the term "damage" refers to falling plaster unless otherwise stated.

In order to comprehend the relationship between vibrations from quarry shots and damaging vibrations, the amplitude and frequency of vibration for all tests made in houses were tabulated for comparison. Acceleration is proportional to the product of the displacement and the square of the frequency for a sinusoidal vibration, and since the structural movements were essentially sinusoidal the accelerations also were computed and tabulated (47).

A study of the computed values for acceleration showed that the average vibration from a quarry shot had an acceleration considerably below that of gravity ( $g=32.2$  feet per second per second), whereas the damaging vibrations from the shaker had accelerations about equal to or greater than that of gravity. Therefore, the acceleration of gravity is a convenient index of the transition from the no-damage to the damage state.

## SUMMARY

Tests of this series gave the following results:

1. Buildings only a few stories high can be made to vibrate in definite modes with a mechanical shaker, as has been done with tall structures. The resonant frequency and the damping can be evaluated, hence the susceptibility to vibration can be determined.
2. Vibrating a building or panel at resonance, within the range of the present tests, is no more destructive than at any other frequency because of the effect of damping inherent in the structure.
3. Panels of buildings can be vibrated by a mechanical shaker so as to cause damage. The resonant frequency and damping effect can be determined, hence the response of the panel to vibration can be evaluated.
4. The effect of vibrations from shaker and quarry shots can be correlated on an empirical basis.
5. For ordinary residential structures the vibration necessary to produce damage is much greater than that resulting from customary quarry blasts.

*Non-exp!*  
*Note*  
*Q=4.5*  
*Fig. 9*

6. Within the range of these field tests an acceleration equal to that of gravity ( $g$ ) can be used as an index of damage.

The use of  $g$  as an index of damage must, however, be tempered by consideration of the physical characteristics of relatively good and poor construction.

#### MATHEMATICAL FORMULAS FOR PREDICTION OF AMPLITUDE

As the field work progressed, expressions were evolved at intervals which, as nearly as possible, represented the trend of the recorded seismic data. The most significant variables influencing the displacement were found to be weight of explosive charge and distance from shot, hence these factors figure largely in all the derived formulas.

In a former publication (28) the authors showed that the amplitude could be expressed as:

$$A = Ke^{.371C}, \quad (3)$$

and

$$A = \frac{a_0}{d^2}, \quad (4)$$

in which

- $A$  = maximum resultant amplitude,
- $K$  = a constant depending on distance from shot,
- $C$  = weight of explosive,
- $a_0$  = a constant depending on weight of charge,
- $d$  = distance from shot,
- $e$  = base of Napierian log (2.72).

It was pointed out that the factor 0.371 in equation 3 was entirely local. Furthermore equation 4 was for one location, thus the usefulness of the results was limited. The implications of the equations are far reaching, however, in that they indicate that assumptions of plane or spherical seismic waves in an extended homogeneous medium cannot be made without modification.

It is demonstrable in physics that the amplitude of a spherical wave in an extended homogeneous medium will vary inversely as the distance and directly as the square root of the energy (or weight of explosive, if it is assumed that the energy is proportional to the weight of explosive). The deviation of equations 3 and 4 (derived from observed data) from the assumption that the amplitude varies as the square root of the weight of explosive charge and inversely as the distance is shown graphically in figure 10.

It is readily seen that an extension of the observed curve in figure 10,  $A$ , to a charge of zero would indicate an amplitude of about 0.00016 inch. As a charge of zero would, of course, result in an amplitude of zero, obviously equation 3 does not represent the true relationship between amplitude and charge except over a limited section (1 to 4 pounds in this example). With a charge of 1 to 4 pounds, however, equation 3 agrees with the observed data. It should be noted that a curve such as the dotted curve of figure 10,  $A$ , may actually represent the conditions governing the observed points plotted on the solid curve. Failure of the observed points to fall directly on the assumed (dotted) curve may be attributed to lack of precision in the observations.

Figure 10,  $B$ , indicates a high degree of initial damping in the observed data. This effect was substantiated by many later readings.

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Examples were given in a previous paper (28) showing how the amplitude of vibration was increased by soil or overburden.

A paper summarizing the results of quarry blasting (43) presented a formula that expressed the amplitude of vibration in terms of weight of explosive charge and distance. Because this formula was

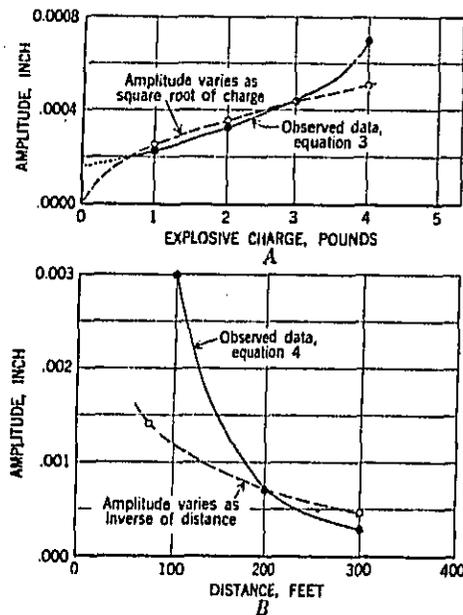


FIGURE 10.—Observed and calculated displacement curves.

based upon a large number of quarry observations, it is especially applicable to quarry blasts. The formula follows:<sup>a</sup>

$$A = \frac{C^{3/4}}{100} (0.07e^{-4.0015d} + 0.001), \quad (5)$$

where  $A$  = maximum resultant amplitude, inches;

$C$  = explosive charge, pounds;

$d$  = distance, feet;

$e$  = base of Napierian log (2.72).

The limits of the formula are:

Charge, 1,000-15,000 pounds; distance, 500-6,000 feet.

Charge, 100-1,000 pounds; distance, 100-6,000 feet.

Charge, 10-100 pounds; distance, 100-1,000 feet.

The limits are necessary for two reasons: The data beyond these limits were scarce and were neglected in deriving the formula, and

<sup>a</sup> See table 6, p. 66.

(2) the formula is purely empirical and reduces to absurdity at extremes of distance. Thus, if the distance is infinite the amplitude, according to the formula, will be  $\frac{C^{3/4}}{100} \times 0.001$ ; if the distance is zero the amplitude will be  $\frac{C^{3/4}}{100} \times 0.071$ . Both figures are obviously false.

The limits give ranges of weight and distance outside of which the formula does not apply. It should be noted, however, that even within these limits opposite extremes of weight and distance tend to give inaccurate results. In general, heavy weights should be combined with the long distances and light weights with short distances. In actual practice, this is usually the case. For example, a building would rarely be within a few hundred feet of a 15,000-pound shot, and the ground vibration of a 10-pound shot more than a few thousand feet away would be of no concern. Briefly, predictions involving shots of 1,000 pounds or more should not be attempted with this formula for distances less than 500 feet or for shots of 100 pounds or less for distances beyond 1,000 feet.

In a former paper evidence was given (48) to show how the amplitude increased on overburden. In general it was found that the amplitudes on rock outcrops were only about one-tenth those observed on overburden for comparable conditions. For convenience the formulas for amplitude were derived for conditions of average overburden. The amplitudes so derived should therefore be decreased about 10 times for rock outcrops and increased about 3 times for deep or abnormally responsive overburden, such as sand-gravel-loam deposits. Amplitudes computed from equation 5 and observed amplitudes were compared with favorable results.

Although the introduction of other variables doubtless would give more accurate results, the difficulty of evaluating the variables and the increased complexity of the formula would seriously limit its usefulness; furthermore, the accuracy of equation 5 was sufficient for most practical purposes.

When controlled tests were made underground (44), conditions were probably as nearly ideal as can be attained in practice. Thus, it was found that the most satisfactory expression for amplitude in terms of charge and distance was similar to the theoretical equation for amplitude of sound in a homogeneous medium; that is, that the amplitude decreased inversely with the distance and was proportional to the square root of the weight of explosive.

A correction in the formula was made for weight-strength of the explosive. Although this designation is not subject to a rigorous definition and probably varies with the method of determination, in these tests it apparently coincided with some physical property of the explosive which influenced the vibrations. This property, which for lack of a better term is called the ingredient weight-strength, is represented in the formula by  $S$ . As the ingredient weight-strength does not differ greatly for the majority of the explosives employed in these tests the correction is minor. For this reason, the equation can be simplified by substituting for  $S$  the average weight-strength of 60 percent, or 0.6, as shown below.

Formula for mine shots:

$$A = \frac{0.035\sqrt{SC}}{D}$$

If  $S=0.6$ ,

$$A = \frac{0.027\sqrt{C}}{D}$$

where  $A$ =maximum single amplitude, inches;  
 $C$ =weight of explosive, pounds;  
 $D$ =distance, feet.

This formula probably is best-adapted to predicting vibrations in mines where records are taken on the same rock stratum as the blast.

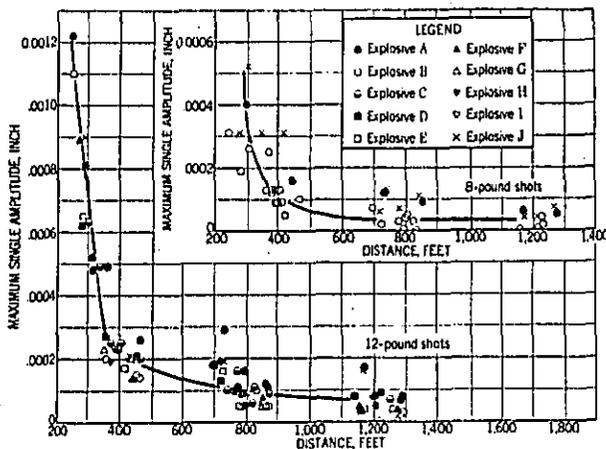


FIGURE 11.—Amplitude-distance curves.

The formula was based upon shots restricted in weight (20 to 200 pounds) and in distance (300 to 1,500 feet); hence the formula should be used only between these limits.

For small weights of explosive (8 and 12 pounds) made under closely controlled conditions, the amplitude varied with the distance, as shown in figure 11. This figure indicates that the amplitude decreased inversely as the 3.0 power of the distance from 200 to 400 feet. From 400 feet to the greatest distance at which records were made—1,300 feet—the amplitude decreased inversely as the distance. This difference illustrates the high damping near the shots.

As the damping in either rock strata or overburden is very rapid at short distances, correct estimation of amplitudes close to a blast is difficult. Table 3 gives a rough idea of the amplitudes in overburden that might be expected 75 feet from a shot.

TABLE 3.—Amplitudes at 75 feet

Weight of explosive, pounds:	Amplitude, inch
1	0.001
5	.01
10	.025
50	.10
100	.16

Throughout these derivations the frequency has not been included, because for a given type of ground the predominant frequency is virtually constant; that is, it does not depend on the amplitude. A difference in type of ground (for example, rock outcrop compared to overburden), however, will produce quite a difference in frequencies. For general use the following frequencies are representative:

	Cycles per second
On outcrops	50
On average overburden	15
On abnormal overburden	5
Residential buildings	10

Abnormal overburden is exceptionally deep overburden (50 feet or more) or overburden composed of a sand-gravel-loam type of deposit.

As the vibrations are generally irregular it should be kept in mind that frequency refers to the predominant frequency. Therefore, sinusoidal conditions can be assumed for computing velocity, energy, etc., only if justified by the seismic records.

## SUMMARY

The amplitudes of vibration from quarry and mine blasting can be predicted through application of the following formulas:

## (1) For quarries:

Charge, 1,000-15,000 pounds; distance, 500-6,000 feet.

Charge, 100-1,000 pounds; distance, 100-6,000 feet.

Charge, 10-100 pounds; distance, 100-1,000 feet.

$$A = \frac{C^{7/3}}{100} (0.07e^{-0.00148d} + 0.001),$$

where  $A$  = maximum resultant amplitude, inches;

$d$  = distance, feet;

$C$  = explosive charge, pounds.

This amplitude is based upon average overburden. For outcrops, divide amplitude by 10. For deep or abnormal overburden, multiply by 3. (The amplitude on the first story of a house may be taken as equal to that on the ground.)

## (2) For mines where the observations are on the same rock stratum:

Shots: 20-200 pounds.

Distances: 300-1,500 feet.

$$A = \frac{0.027\sqrt{C}}{d}$$

## (3) For small shots in mines, the curves of figure 11 can be used.

Shots: 5-15 pounds.

Distance: 200-1,500 feet.

(4) For close shots, an accurate estimate is difficult to make. Table 3 indicates the trend for observations on overburden.

Shots: 1-100 pounds.  
Distance: 75 feet.

#### DETAILS OF TECHNICAL STUDIES

The preceding pages present a chronological story of the research program. Details of the field tests have been published in several progress reports; however, many technical details were omitted. These technical details are essential in providing justification for some of the conclusions reached and are presented herewith.

#### STRUCTURAL RESPONSE TO VIBRATIONS

In order to understand how buildings are affected by vibrations, it is desirable to know the manner or mode of vibration. Numerous configurations are possible, even in the simplest structures. One or more of these configurations may be observed experimentally or deduced theoretically. For experimental observations both buildings (47, 52) and models (19) have been used. For theoretical deductions certain assumptions have to be made as to the physical characteristics of the building.

The assumptions generally reduce the problem to that of a building equivalent to either (1) a uniform homogeneous bar or (2) a series of discrete masses connected by elastic members of known rigidity. Naturally, the choice of assumptions is governed by the type of structure.

#### TRANSLATIONAL MOVEMENT

Vibrations of a homogeneous bar or of a model composed of a series of discrete masses result in displacements characterized by the same general forms. Taking, for example, the simplest case, a building will vibrate in a vertical plane along one of its major axes as shown in figure 12. This is the mode of lowest frequency at which the building will vibrate naturally. The first overtone, a second mode, will result in displacements such as those shown in figure 13; the third mode will be as shown in figure 14; and so on.

Insofar as these modes are characterized by all particles of the building moving in the same line of direction, the term "translational" is applicable and convenient.

In general, for the translational modes, flexure will predominate if the building is tall and slender and shear will predominate if the building is short and wide. Actually, neither stress is entirely absent.

If flexure predominates and the building acts like a uniform bar clamped at one end, the ratios of the overtone frequencies to the fundamental frequency are well-known (55, p. 115) and will be approximately 0.3, 17.6, 34.4, etc. If shear predominates and the building acts like a short, uniform bar, the ratios of the overtone frequencies to the fundamental frequency will be (52, p. 51) approximately 3, 5, 7, etc.

Observations by Jacobsen and Ayre (19) on a model which was designed as a series of discrete masses and which allowed shear and

flexural distortions resulted in ratios of approximately 4, 7, 6, 10.4, etc. Thus, the ratio of frequencies is indicative of the type of distortion.

These translational modes may be excited along either major axis of a building separately or simultaneously.

#### TORSION

A building also may vibrate in torsion. In motion of this type the particles of the building move in arcs (in horizontal planes) about the center of torsion.

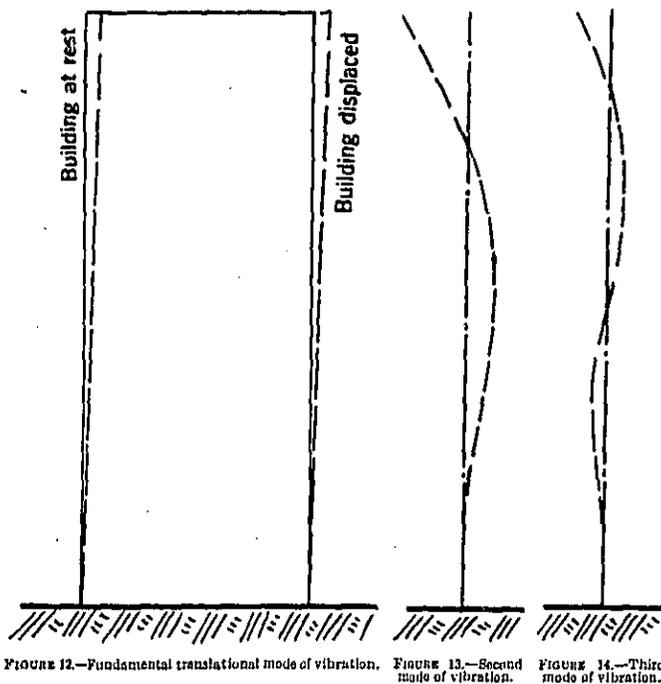


FIGURE 12.—Fundamental translational mode of vibration. FIGURE 13.—Second mode of vibration. FIGURE 14.—Third mode of vibration.

In torsion, as in the translational modes, there is a certain minimum frequency at which the building will vibrate naturally or freely. This is the frequency of the fundamental mode. Figure 15 shows the angular displacements of a building about its main vertical axis, which remains fixed. For the higher modes, figures 13 and 14 are applicable if the displacements shown are considered as representing angular displacements. In torsion the ratios of overtone frequencies to the fundamental mode are 3, 5, 7, etc. (55, p. 153).

## ROTATION

A building may vibrate not only in translation and torsion but also in rotation. In a distortion of this type the horizontal sections (or floors, as the case might be) rotate about horizontal axes. (See fig. 16.) Usually the rotational distortions are neglected in designing earthquake-resistant buildings, either because they are negligible compared to the horizontal distortions or because the vertical strength of the building greatly exceeds the vertical stresses set up. For the same reasons vertical translational vibrations, or what would correspond to longitudinal vibrations in a bar, are neglected.

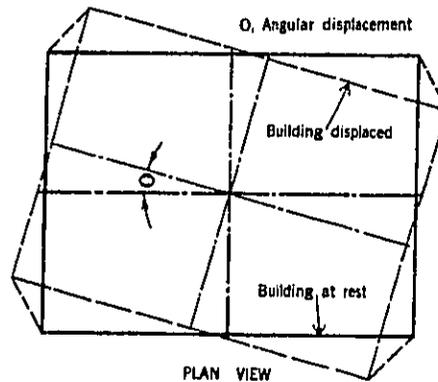


FIGURE 15.—Torsional vibration.

In all modes of vibration yielding of the ground must be considered. This yielding corresponds to yielding of the support of a clamped bar which is equivalent to a lengthening of the bar.

## COUPLED SYSTEMS

In addition to the foregoing modes of vibration, which occur individually or collectively, another phenomenon—that of coupled systems—is possible in the vibration of buildings.

If two or more independent vibrating systems are so placed or linked that energy can be transferred from one to the other, a coupled system results. For example, the ground and a building on the ground form a coupled system; likewise, two adjacent buildings may form a coupled system through the ground. Even a tower on a building and the building itself may form a coupled system. In general, the energy transfers alternately from one system to the other until dissipated by friction and other losses. Naturally, the rapidity and efficiency of the transfer depend on the characteristics of the coupling.

For a particular mode of vibration, as many resonant frequencies will be obtained as there are numbers of coupled systems (31, p. 43). Furthermore, the resulting resonant frequencies are not the resonant frequencies of the individual systems.

SEISMIC EFFECTS OF QUARRY BLASTING

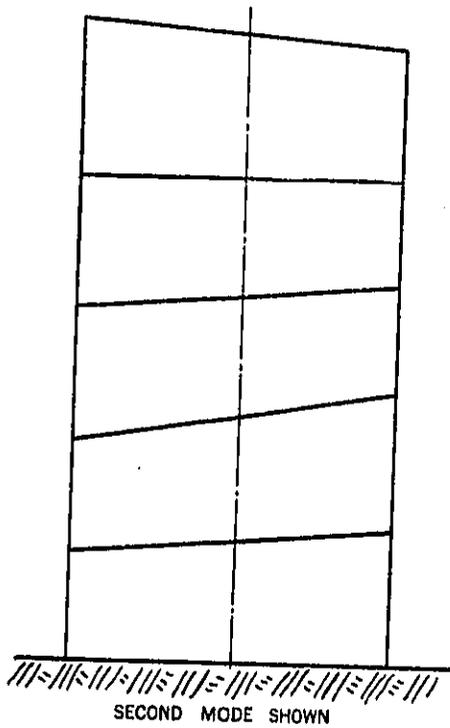


FIGURE 16.—Rotary vibration.

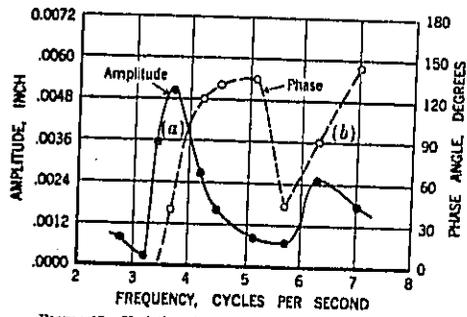


FIGURE 17.—Variation of amplitude and phase with frequency.

Assume two systems, one with a natural frequency of  $f_1$  and the other with a natural frequency of  $f_2$ . If these are coupled to form a third system and if the damping is low, the two resonant frequencies of the coupled system will be, ordinarily, two other frequencies,  $f'$  and  $f''$ , which bear a definite relationship to  $f_1$  and  $f_2$  for the coupling employed.

#### OBSERVED MODES OF VIBRATION

The buildings tested in this investigation presented a unique problem. In virtually all previous investigations (7, 19, 52, 53) tall office buildings were tested, whereas most of the buildings tested by the Bureau were two-story frame structures. Although two-story models had been tested (2) and some investigations had been made on residential dwellings (20, 36), no data were available showing the modes of vibration and correlating them with incoming vibrations such as emanate from quarry blasts.

Most of the buildings tested (47) during this investigation gave clear evidence of response to vibrations in one or more of the fundamental modes. No overtones or higher modes were obtained because the high frequency necessary to excite them was not available in the shaker used.

Typical results can be illustrated best by considering a few actual examples.

#### BUILDING A

Building A was a two-story frame structure with no basement. The plan dimensions were 23 feet east-west by 40 feet north-south and the attic floor was 19 feet above ground. Recording stations were placed on the first, second, and attic floors, and the building was subjected to a vibration of varying frequency by means of the mechanical shaker. Figure 17 shows how the amplitude of the east-west displacement varied with the frequency. Two resonant frequencies, 3.7 and 6.3 cycles, are apparent. The vibration at the lower frequency is characterized by the building swaying as a unit, as shown in figure 18. This is the fundamental translational mode.

While the vibrations were being recorded a continuous record was kept of the phase between shaker force and building displacement. The shaker force and speed were recorded by an electrical impulse which placed an identifying mark on the displacement oscillogram for every revolution of the shaker. The marks corresponded to definite orientation of the unbalanced weights in the shaker and, hence, to a certain position of the force cycle. The electrical impulse necessary to deflect the oscillograph element was obtained by means of the circuit shown in figure 19.

With potentiometer  $R$  (fig. 10), the grid bias of the gas triode is adjusted to a point just below the break-down value of the gas. As the rotating iron bar on the shaft of the shaker passes over the telephone earpiece, a voltage is induced which, when added to the bias voltage, causes the gas in the tube to break down and renders it conducting. Immediately, condenser  $C$ , which has been charged to plate-battery potential, discharges through the tube, a protective resistance, and the oscillograph element or galvanometer. As the condenser discharges the element gives an impulse that records on the oscillogram. The plate voltage cannot remain at the original value

while the tube is conducting because of the high-voltage drop across resistance  $R_2$ . The plate voltage drops so low that the tube gas deionizes and restores tube conditions to the initial state ready for the next impulse. Typical circuit constants are shown in figure 19.

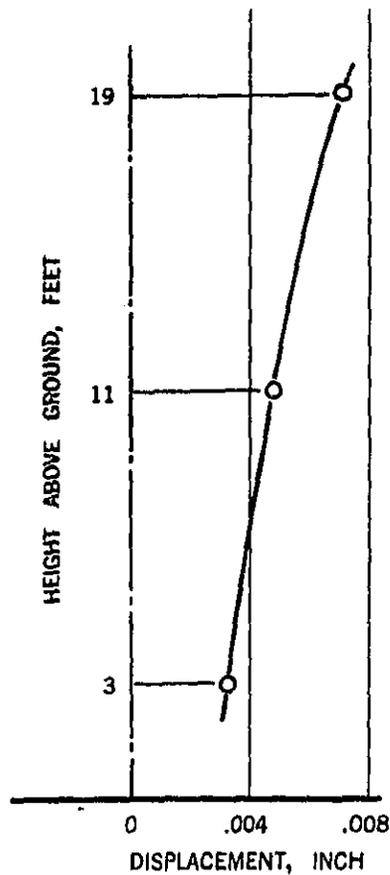


FIGURE 18.—House profile at moment of maximum east-west displacement.

Figure 17 shows that the phase between the force and displacement is  $90^\circ$  at approximately the translational resonant frequency ( $\alpha$ , fig. 17). This was true for all buildings tested.

The vibration that occurred at 6.3 cycles was characterized by the house twisting about a vertical axis. The mode of vibration is shown in figure 15 and is recognized as the fundamental torsional mode.

As the torsional resonant frequency was reached, the phase angle for points near the shaker dropped to a minimum, then passed through

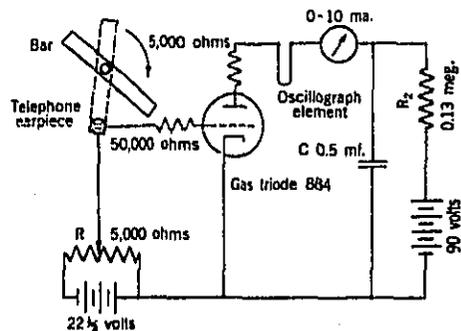


FIGURE 19.—Speed- and phase-measuring circuit.

90° at approximately the torsional resonant frequency (*b*, fig. 17). Points on the opposite side of the center of torsion are, of course, 180° out of phase with points near the shaker.

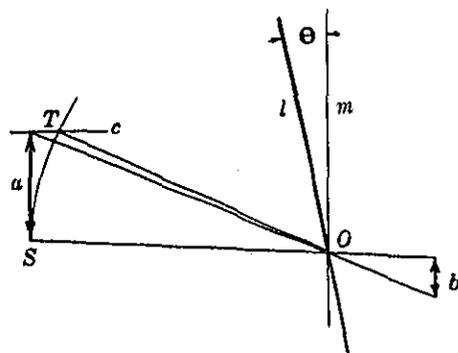


FIGURE 20.—Locus of centers of torsion.

When two displacement recorders respond to vibrations along parallel lines, the locus of centers of torsion can be determined readily by the graphic method shown in figure 20. Assume the magnitude and direction of the recorded vibrations as represented by vectors *a* and *b* (fig. 20). Vector *a* is parallel to *b*; that is, the recorders have been

placed so as to respond to vibrations along parallel lines. The locus is determined as follows:

1. Connect corresponding maxima points of  $a$  and  $b$  and thus determine  $O$ , the point of intersection.
2. Through  $O$  draw line  $m$  parallel to  $a$  and  $b$ .
3. Construct line  $c$  perpendicular to vector  $a$  and through its end point.
4. With  $OS$  as a radius, strike an arc across line  $c$  and thus determine point  $T$ .
5. Construct line  $l$  through  $O$  so that  $\Theta =$  one-half of angle  $SOT$ . Line  $l$  will be the locus of centers of torsion. If two other instruments are used that respond to vibrations along a line other than one parallel to  $m$ , say normal to  $m$ , the center of torsion will be at the intersection of the two loci of centers of torsion for the two sets of recorders.

In most instances the magnitudes of  $a$  and  $b$  will be very small compared to the distance between them, hence the angle  $\Theta$  will be so small that it may be considered zero with no appreciable error. Under these conditions the locus of centers of torsion will be line  $m$ , parallel to  $a$  and  $b$ .

The translational fundamental frequency for the house along the north-south component was 5.3 cycles.

Measurements on this building showed that when the structure was caused to vibrate by slamming a door the predominant motion was translational. The location of the door was not favorable for exciting torsional vibrations, hence the test was negative for torsional vibration. When the building was vibrated by gusts of wind the predominant motion was again translational.

The vibration resulting from blasting in a nearby quarry was irregular but gave a strong indication of torsional vibration. The north end of the house was  $180^\circ$  out of phase with the south end, and the predominant frequency was about 7 cycles, which is comparable to the torsional resonant frequency of 6.3 cycles shown in figure 17.

#### BUILDING B

Building B was a three-story frame house on a concrete foundation. It was situated on a slight knoll in a valley composed of sand-gravel-loam deposits. When this building was vibrated with the shaker the results were similar to those at building A. Both translational and torsional vibrations were excited.

When shots were made at a nearby quarry, however, the house responded with outstanding and regular vibrations at the translational fundamental frequencies. The north-south vibration predominated, and the swaying persisted even after the ground vibrations had subsided. A line through the house and shots was approximately north-south.

The outstanding response at the fundamental frequency was attributed to resonance. The ground frequency along the sand-gravel-loam valley was close to the natural translational north-south frequency of the house.

Although the translational vibrations excited by the shaker and by blasting are quite similar, a minor difference is evident. Figure 21 shows successive positions of the building, plotted at 0.25-second intervals, when the mode was excited from a shot. A definite time lag between floors is evident, resulting in a whipping action. When the same mode was excited by the shaker, there was no appreciable time lag between floors. The position of maximum displacement is

shown in figure 22. The configurations in figures 18 and 22 typify the distortions produced by the shaker.

BUILDING C

Building C was a two-story dwelling with basement on a concrete foundation. The first story was finished with stucco and the second with claphboards.

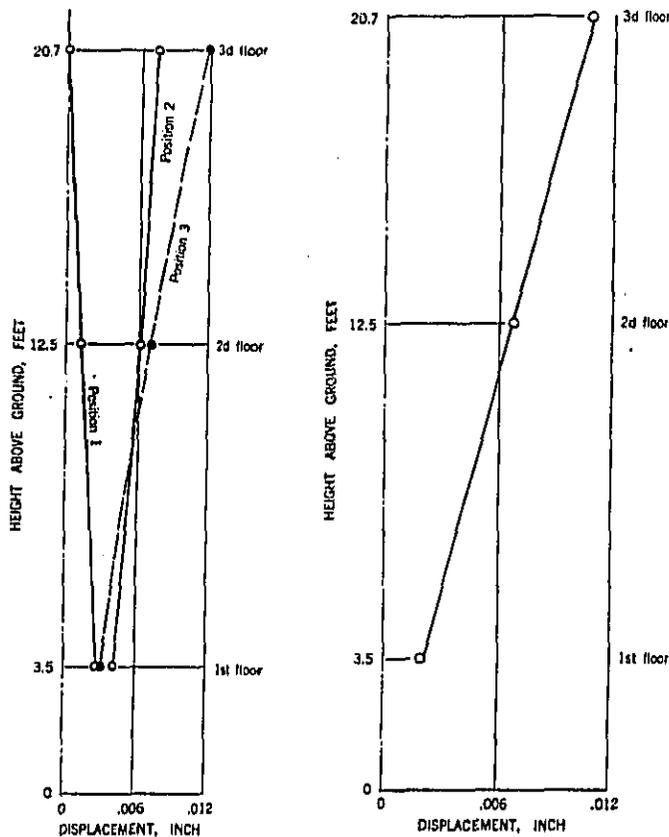


FIGURE 21.—Building deflection from shot vibrations; cellar floor, 3.5 feet below ground.

FIGURE 22.—Building deflection from shaker vibrations; cellar floor, 3.5 feet below ground.

The translational fundamental frequencies were determined with the shaker. The north-south frequency was 5.8 cycles and the east-west frequency 7.0 cycles. There was a suggestion of torsional resonance, but it was not clearly defined.

Shots in a nearby quarry resulted in vibrations, of which a large part were characterized by response of the building as a unit. The motion during part of this period is represented in figure 23. This

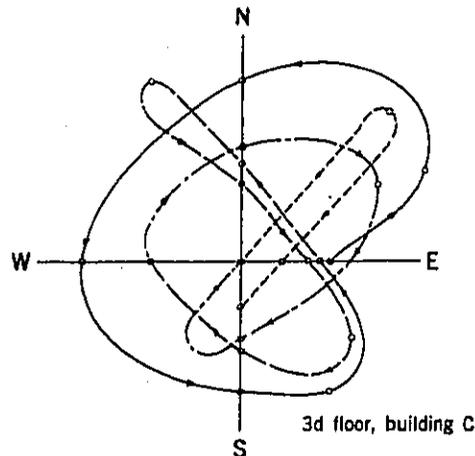


FIGURE 23.—Path of particle resulting from a quarry shot.

motion can be duplicated essentially by adding two horizontal vibrations that are acting simultaneously and are mutually perpendicular. The frequencies of the two vibrations correspond to the two fundamental translational frequencies, 5.8 and 7.0 cycles per second, and

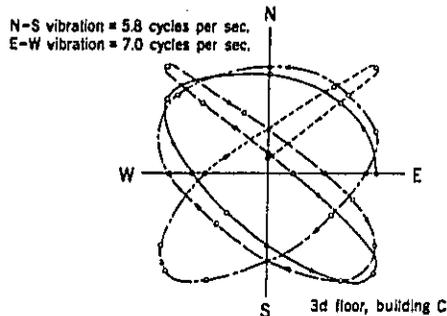


FIGURE 24.—Path of particle resulting from vector addition of two sinusoidal vibrations at right angles.

the vector sum is shown in figure 24. The similarity of this motion to that of figure 23 is apparent.

These and similar observations lead to the following conclusions:

1. Although the disturbance from a shot may result in a complex vibration difficult to analyze, in many instances the form of vibration can be attributed

to either a natural translational mode or a torsional mode or a combination thereof.

2. One mode may be outstanding if resonance occurs, hence a knowledge of these natural modes and the ground frequency is necessary.

3. The distortions observed in these buildings are somewhat intermediate between the configurations (8) for either flexure or shear alone, hence it is assumed that both flexure and shear are important for these fundamental modes.

4. The distortions produced at the natural modes by means of the shaker are very similar to those produced by vibrations from nearby blasting.

#### PANEL VIBRATION

The separation of wall- and floor-panel vibrations from building vibrations not only aids in understanding the phenomena better but is a natural division. A building may sway with large horizontal displacements, yet have relatively little panel motion, or the panel vibration may be intense with little or no building vibration. In general, an external source tends to excite the first (building) type of vibration and an internal source the second (panel).

The first step in understanding panel vibration was to determine the modes in which the panel vibrated.

Without knowledge of the modes of vibration, measurements would be seriously handicapped, because the seismometers could not be located to best advantage, observations at different points could not be correlated, and displacements at points not specifically covered by recorders could not be estimated intelligently.

To obtain information on the influence of the physical constants of floor panels, it is instructive to consider them as homogeneous rectangular plates. Then, according to Timoshenko (49, p. 312),

1. If the thickness is held constant, the frequency of any natural mode varies inversely as the square of the linear dimensions.

2. If all the dimensions (including thickness) are changed in the same proportion, the frequency will vary inversely as the linear dimensions.

3. The frequency varies directly as the square root of the modulus of elasticity and inversely as the square of the density.

For example, if a homogeneous rectangular floor panel was 10 by 10 feet and had a natural frequency of 16 cycles per second, then a similar floor of the same thickness but 20 by 20 feet in area would have a frequency of  $\frac{1}{2} \times 16$ , or 4 cycles per second.

As the physical properties of the panels to be tested were unknown, the agreement between their vibration and the theoretical modes (31, 34, 54, 55) was unknown. For this reason, the modes were determined empirically.

The shaker was used as a source of vibration and usually was mounted in the center of the panel to produce force normal to the plane of the panel. Because the floor panels were easiest to test, most of the observations were made on them. The typical floor panel was composed of wood flooring above, wood joists, wood lath, and plaster below and was about 13 feet square.

A number of floor panels were vibrated at various amplitudes and frequencies, and enough recorders were used so that the configuration of the panel could be established. In some tests a single recorder was employed to "probe" the floor and thus obtain the displacements at a large number of points. Records were made not only of the amplitude but also of the phase between instruments and between instruments and force.

To confirm the instrument records and to establish in more detail the modes of vibration, sand tests were made employing the classic method followed by Chladni (54; 55, p. 164). A thin layer of river

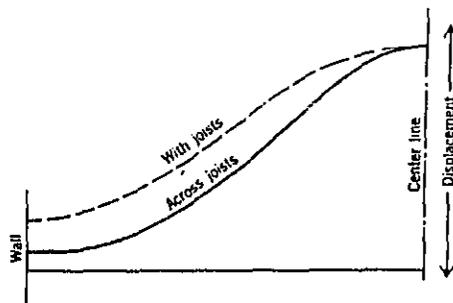


FIGURE 25.—Floor profile at low frequency.

sand was sprinkled uniformly over the whole panel. As the panel vibrated, the sand particles collected along nodal lines or along the boundary of sand-particle activity. In the active regions where the

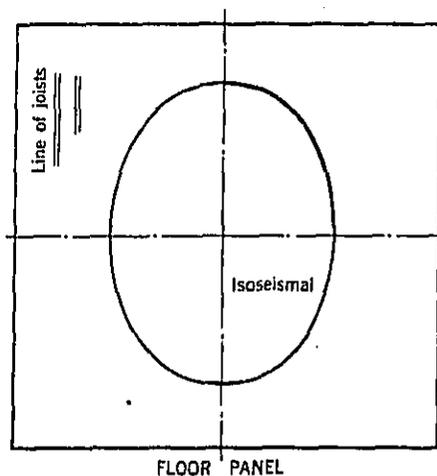


FIGURE 26.—Single-ellipse configuration.

particles "danced," the acceleration of the floor approximately equaled that of gravity or greater.

All floor panels tested were quite similar in size and construction, hence the results were adaptable to a satisfactory general solution, which will be described.

When the panel is vibrated at low frequency the point of largest displacement is at the center, as shown by the profile (fig. 25).

All the points of a particular displacement form a line called an "isoseismal," which in this instance will be an ellipse<sup>1</sup> whose major axis parallels the joists of the floor, as shown in figure 26. All parts of the floor are in phase with one another and with the force; thus when the force is maximum the center is at its maximum displacement, and all other points are at their maxima for this set of conditions.

As the frequency is increased, the displacement at the center of the floor lags the shaker force until at the fundamental resonance the phase lag is 90°. As the frequency is increased still further, the phase angle approaches 180°; that is, when the force is maximum up, the floor is at its maximum displacement down. The outer edges of the

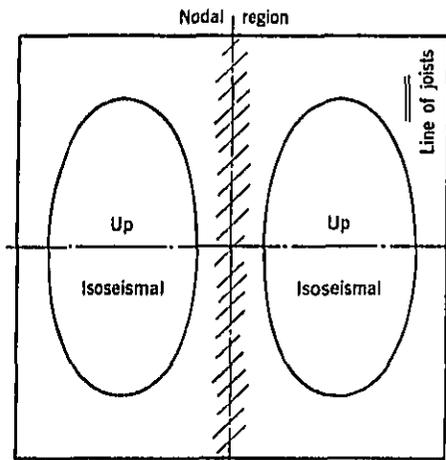


FIGURE 27.—Two-ellipse mode of vibration.

floor tend to lag the center of the floor by an angle that increases as the frequency increases.

After the floor passes through the fundamental resonant frequency (assuming that the forcing frequency is being increased) the amplitude decreases at all points. It is soon observed that the amplitude at the center has decreased more than that on either side of the center, and isoseismals now form two ellipses, as shown in figure 27.

By the time the two-ellipse condition is reached the displacements along the "nodal" region (so-called because of its smaller amplitude) are about 180° behind the force, and the displacements on either side are about 270° behind the force. This means, of course, that the center section of the floor is in quadrature with the elliptical areas, and when the first is a maximum the second is zero. Figure 28, showing four profiles taken at quarter-cycle intervals, illustrates this

<sup>1</sup>Or a figure very nearly the shape of an ellipse.

fact. The two-ellipse condition is essentially a transitory configuration and occupies a small percentage of the frequency range.

As the frequency is increased above the two-ellipse value the outer sections of the floor lag behind the shaker by an angle that soon be-

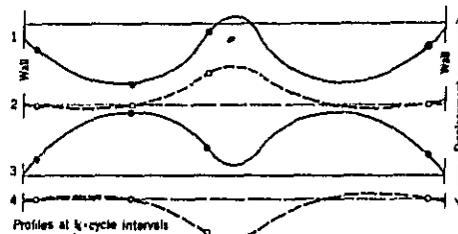


FIGURE 23.—Floor profiles, two-ellipse mode.

comes approximately  $360^\circ$ . The center section of the floor, or nodal region, of the two-ellipse condition remains at a phase lag of  $180^\circ$ . Therefore the center and outer sections are  $180^\circ$  out of phase and thus form three ellipses, as shown in figure 20. As before, the major axes

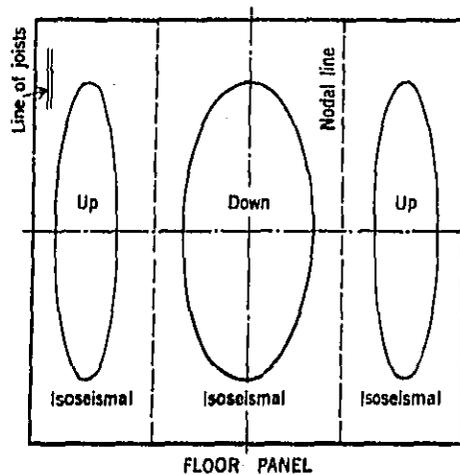


FIGURE 20.—Three-ellipse mode of vibration.

parallel the joists, showing that the panels have different elastic constants along the two major axes. The displacement at the center of the middle ellipse soon becomes maximum because the amplitudes of the outer ellipses drop as the frequency is raised and the amplitude of the center ellipse increases. As the two outer ellipses are out

of phase with the center ellipse, the division lines between are nodal lines of zero amplitude. This is shown by the profile (fig. 30).

To utilize these modes to estimate amplitudes at different points over the floor, three curves were constructed, one for each of the three modes—one-ellipse, two-ellipse, and three-ellipse. Each curve was the average of the envelopes developed for the floors when they vibrated in the same mode. By using the envelopes the maximum amplitudes are determined regardless of phase. The shape of the

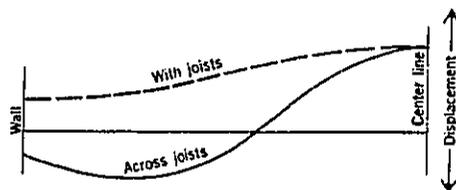


FIGURE 30.—Three-ellipse profile.

envelope does not necessarily coincide with any instantaneous configuration.

It was found that for the one- and three-ellipse modes, one particular shape fitted all distortions quite well, regardless of their amplitude or frequency, within the frequency range for that mode.

As the two-ellipse mode is a transitory configuration the envelope curve (fig. 31) cannot be expressed as a single curve with the same degree of accuracy as that for the one- and three-ellipse modes. This

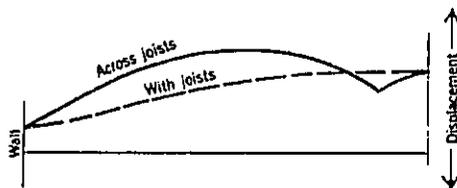


FIGURE 31.—Two-ellipse envelope.

disadvantage is compensated by the comparatively narrow frequency range in which the two-ellipse mode exists.

The three envelopes or master curves are shown in figures 25, 31, and 30 for the one-, two-, and three-ellipse modes, respectively. The use of the curves is quite simple; for example, if the displacement is known at a point halfway from the center of the room to the edge transverse to the joists, the estimated amplitude at the center will be  $M$  times the observed amplitude, where  $M$  equals the ratio (taken from the master curve) of the ordinate at the center to the ordinate at the point in question.

The frequency at which the floor vibrates during transition from a two- to three-ellipse mode is fairly sharply defined and can be determined by noting the frequency at which the amplitude at the center (while increasing) equals that of the two outer ellipses. If this fre-

quency is called  $f$ , then the master curve for the one-ellipse condition is applicable from zero to  $0.7f$  and the master curve for the three-ellipse condition from  $1.1f$  to the highest frequency reached in these tests (about 40 cycles). If the data required for determining  $f$  are not available,  $f$  may be approximated by multiplying the fundamental natural frequency by an empirically determined constant, 2.3, which is the average of a number of tests. If this method is employed the two-ellipse mode (a transition stage) cannot be determined as definitely, and the limits range from  $0.6f$  to  $1.2f$  instead of  $0.7f$  to  $1.1f$ .

The best two-ellipse configuration, similar to that in figure 28, occurs at a frequency of about  $0.75f$ .

It can be seen from a study of figures 25 and 30 that the single- and three-ellipse curves are approximately cosine curves displaced from the horizontal axis. Upon the basis of this fact, two empirical formulas have been developed. For the single-ellipse curve:

$$a = A \left( 0.55 + 0.45 \cos \frac{7.65l}{L} \right);$$

for the three-ellipse curve:

$$a = A \left( 2.3 + 0.77 \cos \frac{8.4l}{L} \right).$$

In both formulas,

$a$  = amplitude of displacement along the center line of the room transverse to the joists, inches;

$A$  = maximum amplitude at center of room, inches;

$l$  = distance of  $a$  from center of room, feet;

$L$  = dimension of room (across joists), feet.

The formulas give good results except near the edge of the floor. Because of this limitation the master curves are preferable to the formulas.

The accuracy of the master curves can be illustrated by comparing a set of computed displacements with the corresponding measured displacements for a series of floor panels. Table 4 gives these displacements.

TABLE 4.—Comparison of observed and computed displacements

Room No.	Frequency, cycles per second	Amplitude, inch		Room No.	Frequency, cycles per second	Amplitude, inch	
		Observed	Computed			Observed	Computed
1.....	10.1	0.085	0.085	3—cont.....	7.8	0.05	0.055
2.....	7.5	.083	.085		17.0	.02	.02
	8.0	.05	.085	4.....	14.3	.07	.07
	9.7	.075	.085		28.0	.04	.04
	24.0	.025	.015		18.4	.12	.12
	10.3	.00	.125		10.0	.00	.00
	19.4	.035	.045	5.....	9.4	.005	.125
	20.0	.035	.045	6.....	14.5	.03	.045
	10.7	.10	.17		28.0	.015	.02
3.....	13.3	.04	.035		12.0	.075	.045
	27.3	.035	.03		29.0	.045	.08
	8.9	.05	.055				

Shaker excitation and seismic vibration differ mainly in that the former is capable of producing a sustained sinusoidal vibration whereas the latter is a transient. A sustained vibration is comparatively easy to measure, whereas a transient vibration that varies with both

space (location) and time requires a large number of pick-ups operating simultaneously. Because of the practical difficulties involved, comparatively little can be said about the mode of vibration of floor panels excited by a transient source; in fact, because the source of excitation is generally of transient nature, it is natural that the vibrated body will not vibrate in a certain characteristic form or mode but rather will vibrate in a complex manner representing the sum of the free and forced vibrations. If the duration of the forced vibration is sufficient, the object may assume a mode correlated with it. If the period of excitation is brief, the object may vibrate as a free body and assume a mode characteristic thereof. If the damping is low, this free vibration may persist for some time after the source of excitation has been removed.

Several observations have been made of floor and wall panels in which the mode of vibration was that of the single ellipse illustrated

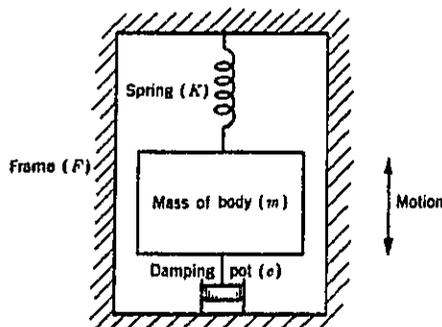


FIGURE 32.—Simple system of vibration.

in figure 26. The vibrations were excited by door slams, truck traffic, people walking, and similar sources.

In one test, made on the floor panel, seven vertical recorders distributed over the panel measured the vibration from a quarry blast. The first part of the record was characterized by a complex motion that smoothed out into a fairly regular vibration with a frequency close to that of the fundamental resonant frequency of the floor as determined by the shaker. The displacements and phase of this section of the record agree closely with the configuration (similar to that shown in fig. 25) obtained with the shaker at the same frequency.

The last part of the record shows all displacements in phase and of approximately the same amplitude, indicating that the panel was moving up and down with no appreciable bending. The frequency of this section was below the resonant frequency.

Thus the record was composed of the following sections:

1. An initial complex motion that was probably a summation of building, panel, free, and forced vibrations.
2. A central part close to resonance, in which the vertical motion was predominantly that of the panel.
3. A final section that showed the building (or walls) moving vertically at a frequency below panel resonance and hence with comparatively little vibration as a panel.

Although the vibration of a panel by use of a shaker at the center may seem to have little connection with the vibration of a panel by movement of its supports (as when excited from an external source such as a blast), actually the two methods of excitation may result in motion of exactly the same type, as pointed out by Den Hartog (13, p. 35). If the floor panel is considered to be a system having a single degree of freedom, with a mass,  $m$ , supported by a spring (spring constant,  $K$ ), and if damping is assumed to be viscous, the resulting motion will be as shown in figure 32.

If it is assumed that the weight (floor) is vibrated by means of the framework  $F$  moving vertically and sinusoidally, the equation of motion can be obtained as follows:

Let  $y$  = relative motion between vibrating body and supporting framework,  
 $x$  = displacement of body from position of rest ( $x=y$  if frame is at rest),  
 $A_0$  = maximum amplitude of frame motion,  
 $\omega$  = angular velocity of driving force,  
 $K$  = spring constant (force per unit elongation),  
 $C$  = damping constant (retarding force per unit velocity),  
 $m$  = mass of body,  
 $p$  = external force,  
 $\theta$  = phase angle between force and displacement,  
 $\omega_0 = 2\pi$  times the undamped resonant frequency of the system,  
 and  
 $t$  = time.

Also let the frame  $F$  move sinusoidally in the form  $A_0 \sin \omega t$ . Then

$$y = A_0 \sin \omega t - x = \text{spring extension}, \quad (6)$$

Therefore,

$$\text{spring force} = Ky = K(A_0 \sin \omega t - x).$$

If viscous damping is assumed,

$$\text{damping force} = Cy = C(A_0 \omega \cos \omega t - \dot{x}).$$

As there is no external force acting on mass  $m$ , the spring inertia and damping forces must be in equilibrium, or

$$m\ddot{x} = Ky + Cy \quad (\text{inertia force opposed to spring and damping forces}).$$

Substituting,

$$\begin{aligned} m\ddot{x} - K(A_0 \sin \omega t - x) - C(A_0 \omega \cos \omega t - \dot{x}) &= 0, \\ m\ddot{x} - KA_0 \sin \omega t + Kx - CA_0 \omega \cos \omega t + C\dot{x} &= 0, \\ m\ddot{x} + C\dot{x} + Kx &= KA_0 \sin \omega t + CA_0 \omega \cos \omega t. \end{aligned} \quad (7)$$

From equation 6, however,

$$\begin{aligned} x &= A_0 \sin \omega t - y, \\ \dot{x} &= A_0 \omega \cos \omega t - \dot{y}, \\ \ddot{x} &= -A_0 \omega^2 \sin \omega t - \ddot{y}. \end{aligned}$$

Substituting in equation 7:

$$-m\ddot{y} - mA_0 \omega^2 \sin \omega t - C\dot{y} + CA_0 \omega \cos \omega t - Ky + KA_0 \sin \omega t = KA_0 \sin \omega t + CA_0 \omega \cos \omega t,$$

or

$$m\ddot{y} + C\dot{y} + Ky = mA_0 \omega^2 \sin \omega t.$$

This equation is the same as that for a mass supported from a fixed frame,  $F$ , and acted upon by a force,  $p$ , where:

$$y = x \quad (\text{the absolute motion}) \quad \text{and} \quad p = mA_0 \omega^2 \sin \omega t, \quad \text{or} \quad p_{\max} = mA_0 \omega^2.$$

The solution of the equation of motion for a mass acted on by force  $p$  is well-known (Sec. 37) and is:

$$x = \frac{p \sin \theta}{C \omega} \cos(\omega t - \theta). \quad (8)$$

As the maximum value that the cosine term can attain is 1, maximum  $x$  will be:

$$x_{\max} = \frac{p \sin \theta}{C \omega}.$$

Substituting for  $p$  the value for  $p_{\max}$ , just found:

$$x_{\max} = \frac{m A_0 \omega^2 \sin \theta}{C \omega} = \frac{m A_0 \omega \sin \theta}{C},$$

where,

$$\sin \theta = \frac{C \omega}{m \sqrt{(\omega_0^2 - \omega^2)^2 + \frac{C^2 \omega^2}{m^2}}}$$

Thus,

$$x_{\max} = y_{\max} = A_0 \frac{\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \frac{C^2 \omega^2}{m^2}}}, \quad (9)$$

which is the equation of motion in terms of the maximum displacement and the frequency of vibration of the framework, the natural frequency of the weight (floor) system, and the damping.

If, however, the same system (fig. 32) is caused to vibrate by a shaker type of force (force that varies as the square of the frequency) acting directly on the mass (floor), the following derivation holds:

$$x = \frac{p \sin \theta}{C \omega} \cos(\omega t - \theta). \quad (8)$$

The force is the centrifugal force of the shaker, thus

$$p = m_1 r_1 \omega^2,$$

where  $m_1$  = mass of unbalanced weight, and  $r_1$  = radius of unbalanced weight.

Substituting for  $p$  in equation 8,

$$x = \frac{m_1 r_1 \omega \sin \theta}{C} \cos(\omega t - \theta),$$

$$x_{\max} = \frac{m_1 r_1 \omega \sin \theta}{C},$$

but, as before

$$\sin \theta = \frac{C \omega}{m \sqrt{(\omega_0^2 - \omega^2)^2 + \frac{C^2 \omega^2}{m^2}}}$$

Therefore,

$$x_{\max} = \frac{m_1 r_1}{m} \frac{\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \frac{C^2 \omega^2}{m^2}}}$$

or let

$$m_0 = \frac{m_1 r_1}{m},$$

then

$$x_{\max} = m_0 \frac{\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + \frac{C^2 \omega^2}{m^2}}}$$

which is in the same form as equation 9, obtained for the moving support.

With both types of vibration the spring represents the elasticity of the floor and the damping the frictional forces.

Several points can be advanced to show that for practical purposes the panels tested agree with the assumed conditions—single degree of freedom, restoring force proportional to displacement, constant mass, and viscous damping (damping proportional to the velocity).

1. It is known that a wall or floor panel is much more resistant to displacements in its plane than displacements normal to its plane. Thus, if the force is applied normal to the plane the panel has, for practical purposes, but one degree of freedom.

2. The resonance curves recorded in these tests are of the shape obtained with a linear spring system (that is, the spring constant does not depend on the displacement). A nonlinear system will distort the resonance curve because the natural frequency is dependent on the amplitude.

3. It is obvious that, in common with virtually all mechanical systems, the mass is constant in these tests.

4. The choice of viscous damping is justified by the fact that the observed phase and amplitude curves both agree fairly well with comparable curves constructed from the equation that assumes viscous damping. If, for example, the damping were dry damping, which is independent of the magnitude of the velocity but which opposes it, the phase angle would show a discontinuous jump at resonance (*IS*, p. 361) rather than passing through resonance in the manner shown in figure 17. Furthermore, the displacements at resonance, except those of extreme amplitude (0.25 inch or more), were found to be approximately proportional to the shaker force, which indicates that the damping could be considered viscous.

#### SUMMARY

1. Both buildings and panels in buildings can vibrate in definite modes which may be excited either by mechanical means or by quarry blasting. Floor panels were shown to have definite modes of vibration characterized by elliptical isoseismals. It was observed that the construction (direction of joists) determined the major axes of the ellipses. Furthermore, it was shown that the mode of vibration depends on the frequency.

2. Under the conditions of these tests, amplitudes of vibration can be estimated accurately for vibrating floor panels by empirical formulas developed in this chapter.

3. Vibrations produced by a shaker that induces a force proportional to the square of the frequency have the same type of motion in a simple system as that produced in the same system by movement of its supports.

For example, when a floor panel is excited by a shaker at the center, it moves in the same manner as when excited by vertical movement of the supporting walls, because, for practical purposes, the panel is a simple system.

4. Vibrations of buildings may be complicated by overlapping of modes or by coupling.

#### DAMPING

If a body is set in vibration and the driving force removed, the vibration will die out in time. The factor responsible for the decline and final cessation of vibration is termed "damping." Damping may be attributed to a number of causes, including internal friction inherent in the vibrating body, external friction, and air resistance.

The response of a vibrating structure at resonance depends largely on the amount of damping. A number of methods can be employed to determine the damping factor for a simple<sup>10</sup> vibrating body. Among these are methods that use either the relationship between phase and frequency or that between amplitude and frequency. As damping exerts its greatest influence at resonance, it is more accurate to measure it at or near the resonant frequency.

The damping factor may be determined through the use of the phase-frequency curve obtained with the mechanical shaker. This factor was evaluated in terms of the slope of the phase-frequency curve at resonance, as follows:

The expression for the phase angle between force and displacement, according to Wood (55, p. 37), is:

$$\phi = \tan^{-1} 2K \frac{p}{n^2 - p^2} \tag{10}$$

where  $\phi$  = phase angle,  
 $K$  = damping coefficient,  
 $p$  = forced angular velocity,  
 $n$  = natural angular velocity.

If the phase angle is plotted against the forced angular velocity ( $2\pi \times$  forced frequency), the slope at any point on the curve from elementary calculus (16, p. 42) will be  $\frac{\delta\phi}{\delta p}$ .

Substituting equation 10 for  $\phi$  the slope will be

$$\frac{\delta\phi}{\delta p} = \frac{\delta \left( \tan^{-1} 2K \frac{p}{n^2 - p^2} \right)}{\delta p} = \frac{2K(n^2 + p^2)}{(n^2 - p^2)^2 + (2Kp)^2}$$

When the forced frequency equals the natural frequency,  $p = n$ , so the slope at the natural frequency is

$$\frac{\delta\phi}{\delta p|_{p=n}} = \frac{4Kn^2}{4K^2n^2} = \frac{1}{K}$$

Thus, the slope at resonance equals the reciprocal of the damping coefficient. This equation provides a convenient method of determining damping from the phase-frequency curve.

If, however, the amplitude-frequency curve from the shaker tests is used a different method applies. The sharpness of resonance is measured by noting the deviation of the frequency from resonance necessary to cause the amplitude to drop a certain percentage. The damping can then be evaluated by an expression derived as follows:

From equation 9 the amplitude, in terms of the natural frequency, forced frequency, and damping, is:

$$A = m_0 \frac{r^n}{\sqrt{(n^2 - p^2)^2 + 4K^2p^2}}$$

in which  $A$  = amplitude of displacement,  
 $p$  = forced angular velocity,  
 $n$  = natural or free angular velocity,  
 $K$  = damping coefficient,  
 $m_0$  = a constant depending on the mass vibrated, the unbalanced mass of the vibrator, and the eccentricity of the unbalanced mass.

<sup>10</sup> Single degree of freedom, viscous damping, and simple harmonic motion assumed.

If the damping factor is small compared to the natural frequency, then the maximum amplitude will occur when the forced frequency coincides with the natural frequency, that is, when  $p=n$ . Therefore

$$A_{\max.} = m \frac{n}{2K^2}$$

in which  $A_{\max.}$  = maximum amplitude, and

$$\frac{A}{A_{\max.}} = \frac{2Kp^2}{n\sqrt{(n^2-p^2)^2 + 4K^2p^2}}$$

If  $A$  is assumed to be  $\frac{1}{2}(A_{\max.})$ , then  $\frac{A}{A_{\max.}} = \frac{1}{2}$ .

Substituting,

$$\frac{1}{2} = \frac{2Kp^2}{n\sqrt{(n^2-p^2)^2 + 4K^2p^2}}$$

which reduces to

$$K = \pm \frac{n(n^2-p^2)}{2p\sqrt{4p^2-n^2}}$$

where  $p$  is the angular velocity at which the amplitude is one-half the maximum and  $K \ll n$ .

With this expression the damping constant can be evaluated from the amplitude-frequency curve provided, of course, the necessary conditions of simple harmonic motion, viscous damping, etc., exist.

#### SUMMARY

The damping, an important physical characteristic of any structure, may be determined from either the phase-frequency or the amplitude-frequency graph by simple expressions derived in the preceding analyses.

#### STRUCTURAL DAMAGE FROM VIBRATION

A common complaint near blasting operations is that plaster has been cracked. Because of this and because cracked plaster is associated with the initial stages of structural damage from earthquakes—intensity 5 and 6 of the Modified Mercalli Intensity Scale of 1931 (50)—it is convenient to utilize the failure of plaster in determining an index of damage.

It is well-known from the studies of others (4, 5, 35) that plaster may crack from many causes other than vibration. The types and causes of defects are explained and described in detail in chapter 10 of National Bureau of Standards Circular 151 (51). A comprehensive list of reasons why walls and ceilings crack is given in Monthly Service Bulletin 44 of the Architects' Small House Service Bureau of the United States, Inc. (40) and is reproduced here because of its general interest.

Forty reasons why walls and ceilings crack:

- Building a house on a fill.
- Failure to make the footings wide enough.
- Failure to carry the footings below the frost line.
- Width of footings not made proportional to the loads they carry.
- The posts in the basement not provided with separate footings.
- Failure to provide a base raised above the basement floor line for the setting of wooden posts.

Not enough cement used in the concrete.  
 Dirty sand or gravel used in the concrete.  
 Failure to protect beams and sills from rotting through dampness.  
 Settling floor joists one end on masonry and the other on wood.  
 Wooden beams used to support masonry over openings.  
 Mortar, plaster, or concrete work allowed to freeze before setting.  
 Braces omitted in wooden walls.  
 Sheathing omitted in wooden walls (excepting in "back-plastered" construction).  
 Drainage water from roof not carried away from foundations.  
 Floor joists too light.  
 Floor joists not bridged.  
 Supporting posts too small.  
 Cross beams too light.  
 Subflooring omitted.  
 Wooden walls not framed so as to equalize shrinkage.  
 Poor materials used in plaster.  
 Plaster applied too thin.  
 Lath placed too close together.  
 Lath run behind studs at corners.  
 Metal reinforcement omitted in plaster at corners.  
 Metal reinforcement omitted where wooden walls join masonry.  
 Metal lath omitted on wide expanses of ceiling.  
 Plaster applied directly on masonry at chimney stack.  
 Plaster applied on lath that are too dry.  
 Too much cement in the stucco.  
 Stucco not kept wet until set.  
 Subsoil drainage not carried away from walls.  
 First coat of plaster not properly keyed to backing.  
 Floor joists placed too far apart.  
 Wood beams spanned too long between posts.  
 Failure to use double joists under unsupported partitions.  
 Too few nails used.  
 Rafters too light or too far apart.  
 Failure to erect trusses over wide wooden openings.

It should be noted, however, that even when failure of plaster is taken as the criterion, damage is not sharply defined but gradual. Visually, the initial indication of damage is the extension of old cracks or dust falling from them. As the severity of vibration is increased new fine cracks are formed, and the plaster may flake or spall slightly. A further increase in the severity of vibration causes more plaster to crack and finally large areas to drop.

Numerous other factors already mentioned, such as faulty construction, aging, settling, and shrinkage, may cause damage and thereby prohibit determination of a unique destructive index that is applicable in all instances, but these factors do not exclude determination of an index that is applicable in most instances and is based upon a large amount of field data.

No damage resulted in any of the blasting operations investigated over the 5-year period, although hundreds of observations were made at about 30 quarries or mines. Hence, all blasting data, except those for special test shots (45, 46), gave no evidence of damage; however, the shaker produced damaging vibrations as well as nondamaging vibrations. Therefore the procedure was to determine the border line between damage and no damage for the shaker tests and correlate the shot data with the shaker observations.

#### INDEX OF DAMAGE

Figure 33 indicates the distribution of data obtained from 160 shaker tests of 16 ceiling panels in 6 buildings about the damage

border line. It can be seen that the solid line delineates a "damage" region. Thus the mathematical equation of the line determines the destructive index. The equation of this line represents (approximately) the acceleration of gravity; that is, every point on the line

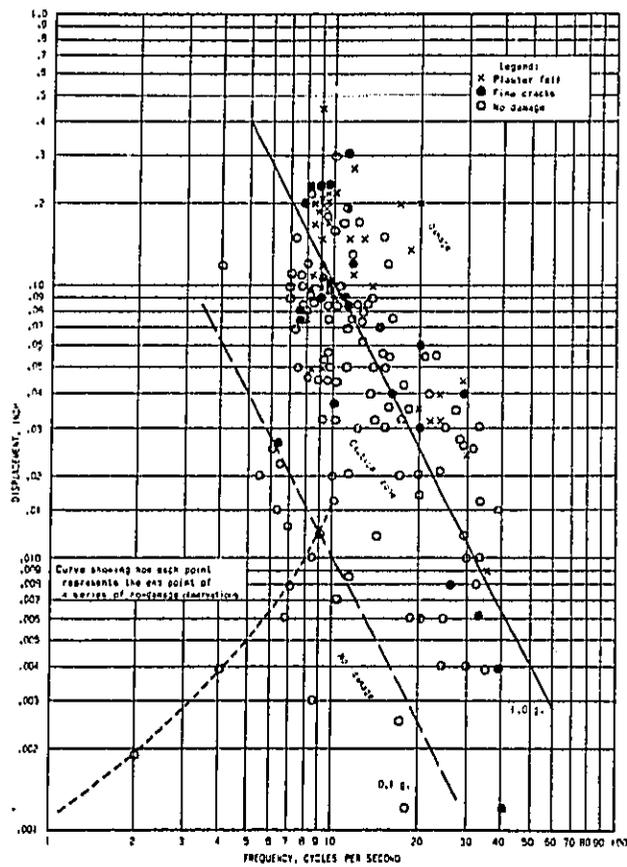


FIGURE 33.—Damage border line.

represents an amplitude at a certain frequency, and the values of these factors are such that the product of  $4\pi^2$  times the frequency squared and the amplitude (in feet) is  $32.2 \pm$  feet per second squared. The dashed line, parallel to the full line, also represents an acceleration, whose value, however, is 3.2 feet per second squared.

It is convenient to express the acceleration in proportional parts of the acceleration of gravity, for example, as one-tenth gravity (0.1 *g*). The acceleration can then be computed<sup>11</sup> from the equation:

$$a = \frac{4\pi^2 f^2 A}{12g}$$

in which *a* = acceleration, in terms of acceleration of gravity;  
*f* = frequency, cycles per second;  
*A* = amplitude, inches;  
*g* = acceleration of gravity, feet per second<sup>2</sup> = 32.2.

Thus,

$$a = \frac{4\pi^2 A}{12 \times 32.2} \approx 0.1 f^2 A. \tag{11}$$

From this equation it can be seen that an amplitude of 0.1 inch at 10 cycles gives an acceleration equal to that of gravity.

$$a \approx 0.1 \times 100 \times 0.1 = 1.0 g.$$

When all the data (fig. 33) were considered an acceleration of 1.0 *g* proved to be the best index of damage. Slight damage, or the preliminary phases of damage, fell within 0.1 and 1.0 *g*.

For comparison the maximum accelerations from 43 quarry shots were computed. These shots were made at 10 quarries and were recorded in 14 buildings. Table 5 gives the data from which the accelerations were computed and figure 34 their frequency of occurrence. The greatest acceleration recorded was 0.1 *g* and the lowest 0.0002 *g*. The typical or most frequent value was 0.01 *g*.

The points of maximum acceleration were selected by study of the oscillograph records. Vibrations of very low amplitude and short duration were neglected, even though the accelerations may have been high, because these conditions were noticeable in the records of many tests that did not cause damage. Although the average of the frequencies tabulated in table 5 is about 10, a few high frequencies occur; however, these high frequencies are generally associated with small displacements, so that the resulting acceleration is low.

TABLE 5.—Accelerations in buildings

Building	Test No.	Sismometer orientation	Displacement, inch	Frequency, cycles per second	Acceleration †
A.....	725	Horizontal.....	0.0002	5	0.0005
	726	do.....	.0003	4	.0005
	727	do.....	.0002	5	.0005
	728	do.....	.0003	5	.0005
	729	do.....	.0002	4	.0003
	730	do.....	.0002	5	.0005
	731	do.....	.0003	6	.001
B.....	732	Vertical.....	.0001	11	.001
	733	Horizontal.....	.0001	4	.0002
	734	do.....	.0003	6	.001
C.....	419	do.....	.0006	0	.002
	.....	do.....	.00015	33	.02
D.....	162	do.....	.001	7	.02
	163	Vertical.....	.002	25	.1
	164	do.....	.0001	30	.009
	165	do.....	.0002	30	.02
	166	do.....	.002	17	.06

† In decimal parts of gravity.

‡ Assuming a sinusoidal vibration.

TABLE 5.—Accelerations in buildings—Continued

Building	Test No.	Seismometer orientation	Displacement, inch	Frequency, cycles per second	Acceleration
E.....	192b	Vertical.....	0.0003	22	0.01
		Horizontal.....	.0006	7	.001
	192c	Vertical.....	.0002	20	.006
		Horizontal.....	.0004	7	.002
F.....	197	Vertical.....	.025	5	.02
G.....	211	Vertical.....	.033	12	.04
H.....	214	Vertical.....	.023	15	.07
		Horizontal.....	.006	12	.09
I.....	223	Vertical.....	.001	30	.00
		Horizontal.....	.0004	17	.02
J.....	874	Vertical.....	.0015	17	.05
		Horizontal.....	.001	20	.04
	875	Vertical.....	.0005	7	.002
		Horizontal.....	.0017	15	.05
K.....	878	Vertical.....	.0008	20	.03
		Horizontal.....	.0008	6	.002
L.....	870	Vertical.....	.0015	11	.02
		Horizontal.....	.002	11	.02
	872	Vertical.....	.0031	9	.02
		Horizontal.....	.0030	9	.03
	855	Vertical.....	.0008	11	.002
		Horizontal.....	.0039	5	.01
	818	Vertical.....	.0030	11	.05
		Horizontal.....	.0023	9	.02
	827	Vertical.....	.0007	23	.04
		Horizontal.....	.009	6	.00
	828	Vertical.....	.002	6	.007
		Horizontal.....	.0025	6	.02
	820	Vertical.....	.001	6	.004
		Horizontal.....	.0015	5	.004
	832	Vertical.....	.001	9	.008
		Horizontal.....	.0008	13	.01
	833	Vertical.....	.001	7	.004
		Horizontal.....	.0004	8	.005
	819	Vertical.....	.0004	16	.02
		Horizontal.....	.0005	9	.004
	818	Vertical.....	.001	5	.04
		Horizontal.....	.002	11	.02
M.....	820	Vertical.....	.0017	7	.008
		Horizontal.....	.0007	33	.08
	819	Vertical.....	.005	4	.003
		Horizontal.....	.004	5	.01
	817	Vertical.....	.0016	22	.08
		Horizontal.....	.0023	6	.008
	816	Vertical.....	.004	6	.01
		Horizontal.....	.0032	16	.08
	815	Vertical.....	.0032	5	.008
		Horizontal.....	.004	6	.01
	815	Vertical.....	.0013	20	.05
		Horizontal.....	.015	5	.04
	815	Vertical.....	.0027	6	.01
		Horizontal.....	.001	25	.08
	815	Vertical.....	.011	5	.03
		Horizontal.....	.0026	5	.009
N.....	812	Vertical.....	.0007	22	.03
		Horizontal.....	.009	7	.04
O.....	846	Vertical.....	.012	11	.1
		Horizontal.....	.01	8	.04
	843	Vertical.....	.005	13	.08
		Horizontal.....	.0025	11	.03
	804	Vertical.....	.0052	6	.03
		Horizontal.....	.005	14	.10
	804	Vertical.....	.01	8	.08
		Horizontal.....	.008	8	.05
		Vertical.....	.003	12	.04

Figure 34 shows clearly that vibrations from blasting are well below the index of damage. The vibrations recorded in special tests (45), in which a structure was damaged by abnormal blasting, indicated accelerations of the order of 1.0 g at the damage point, hence they further confirm the adequacy of the index derived from the shaker tests.

The classification of the index of damage as an acceleration does not mean that acceleration is the cause of damage but simply that the use of acceleration as an index will give a workable method for determining the imminence of damage.

Tables 6 and 7 facilitate estimation of damaging vibration in terms of weight of explosive charge and distance. These tables, based upon empirical data, permit quick calculation of whether a certain combination of weight and distance is likely to produce an acceleration of dangerous magnitude. Corrections are introduced (as footnotes to the tables) to compensate for differences in overburden.

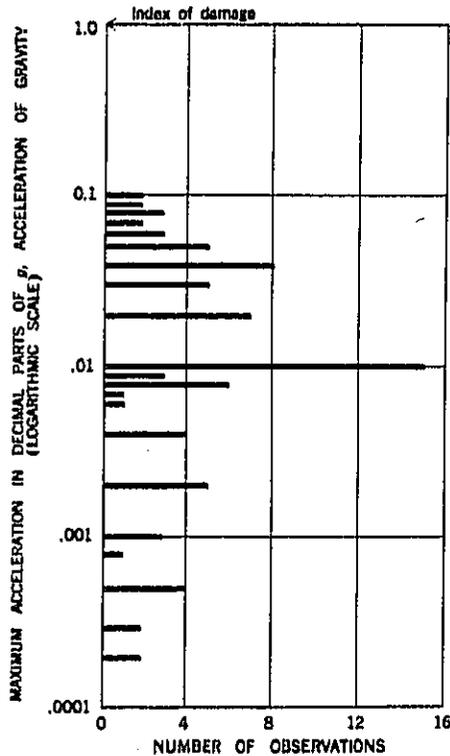


FIGURE 31.—Frequency of occurrence of accelerations.

Two sample computations illustrate the use of the table. Assume that a quarry operator plans to fire a blast of 300 pounds of explosive. The nearest dwelling is situated on average overburden 2,000 feet from the shot. Table 6 indicates that 300 pounds at 2,000 feet should produce a displacement of 0.0022 inch. In table 7 the nearest displacement to 0.0022 inch is 0.002. This figure, combined with the typical residential structure frequency of 10 cycles, represents an acceleration of 0.02 that of gravity. (See table 7.) Damage would not occur because the acceleration, 0.02 *g*, is in the safe region.

TABLE 6.—Displacement<sup>1</sup> for various weights of explosive, inch

Weight of explosive, pounds	Distance, feet														
	100	200	300	400	500	600	700	800	900	1,000	2,000	3,000	4,000	5,000	6,000
10	0.0029	0.0023	0.0022	0.0019	0.0016	0.0014	0.0013	0.0011	0.0010	0.0009	0.0008	0.0007	0.0006	0.0005	0.0004
20	0.0045	0.0039	0.0034	0.0030	0.0026	0.0023	0.0020	0.0017	0.0015	0.0013	0.0011	0.0009	0.0008	0.0007	0.0006
30	0.0059	0.0052	0.0045	0.0039	0.0034	0.0029	0.0025	0.0022	0.0019	0.0016	0.0014	0.0011	0.0009	0.0008	0.0007
40	0.0072	0.0063	0.0054	0.0047	0.0041	0.0036	0.0032	0.0027	0.0023	0.0020	0.0017	0.0014	0.0011	0.0009	0.0007
50	0.0085	0.0074	0.0063	0.0055	0.0045	0.0042	0.0037	0.0032	0.0028	0.0024	0.0021	0.0017	0.0014	0.0011	0.0009
60	0.0098	0.0086	0.0073	0.0063	0.0053	0.0047	0.0042	0.0036	0.0031	0.0027	0.0023	0.0019	0.0016	0.0013	0.0010
70	0.0110	0.0096	0.0081	0.0070	0.0059	0.0052	0.0047	0.0041	0.0035	0.0030	0.0026	0.0022	0.0018	0.0015	0.0012
80	0.0121	0.0105	0.0088	0.0076	0.0064	0.0056	0.0050	0.0044	0.0038	0.0033	0.0028	0.0024	0.0020	0.0016	0.0013
90	0.0132	0.0114	0.0095	0.0082	0.0069	0.0061	0.0055	0.0048	0.0042	0.0036	0.0031	0.0026	0.0022	0.0018	0.0014
100	0.0143	0.0123	0.0103	0.0089	0.0075	0.0066	0.0060	0.0053	0.0046	0.0040	0.0034	0.0029	0.0024	0.0020	0.0016
200	0.0225	0.0188	0.0156	0.0134	0.0112	0.0100	0.0094	0.0087	0.0079	0.0071	0.0062	0.0053	0.0045	0.0038	0.0032
300	0.0315	0.0252	0.0210	0.0181	0.0151	0.0131	0.0125	0.0117	0.0109	0.0101	0.0091	0.0081	0.0072	0.0063	0.0055
400	0.0395	0.0315	0.0261	0.0225	0.0191	0.0167	0.0151	0.0143	0.0135	0.0127	0.0117	0.0107	0.0097	0.0088	0.0079
500	0.0465	0.0365	0.0300	0.0258	0.0220	0.0191	0.0173	0.0164	0.0155	0.0146	0.0136	0.0126	0.0116	0.0106	0.0096
600	0.0525	0.0405	0.0329	0.0280	0.0238	0.0204	0.0184	0.0174	0.0164	0.0154	0.0144	0.0134	0.0124	0.0114	0.0104
700	0.0575	0.0445	0.0359	0.0303	0.0257	0.0219	0.0197	0.0186	0.0175	0.0164	0.0154	0.0144	0.0134	0.0124	0.0114
800	0.0625	0.0485	0.0389	0.0327	0.0277	0.0234	0.0210	0.0198	0.0187	0.0176	0.0165	0.0155	0.0144	0.0134	0.0124
900	0.0675	0.0525	0.0419	0.0351	0.0300	0.0253	0.0227	0.0214	0.0202	0.0191	0.0180	0.0170	0.0160	0.0150	0.0140
1,000	0.0725	0.0565	0.0449	0.0375	0.0321	0.0270	0.0242	0.0228	0.0216	0.0204	0.0193	0.0182	0.0172	0.0162	0.0152
2,000	0.1100	0.0840	0.0650	0.0540	0.0450	0.0380	0.0330	0.0290	0.0260	0.0230	0.0200	0.0170	0.0140	0.0110	0.0080
3,000	0.1350	0.1020	0.0790	0.0650	0.0540	0.0460	0.0400	0.0350	0.0310	0.0270	0.0230	0.0190	0.0150	0.0110	0.0080
4,000	0.1550	0.1170	0.0910	0.0740	0.0610	0.0520	0.0450	0.0390	0.0340	0.0300	0.0260	0.0220	0.0180	0.0140	0.0100
5,000	0.1700	0.1270	0.0970	0.0780	0.0640	0.0540	0.0470	0.0410	0.0360	0.0310	0.0270	0.0230	0.0190	0.0150	0.0110
6,000	0.1850	0.1370	0.1030	0.0820	0.0670	0.0560	0.0490	0.0430	0.0380	0.0330	0.0290	0.0250	0.0210	0.0170	0.0130
7,000	0.1950	0.1450	0.1090	0.0870	0.0710	0.0600	0.0530	0.0470	0.0410	0.0360	0.0310	0.0270	0.0230	0.0190	0.0150
8,000	0.2050	0.1530	0.1140	0.0910	0.0740	0.0620	0.0550	0.0490	0.0430	0.0380	0.0330	0.0290	0.0250	0.0210	0.0170
9,000	0.2100	0.1570	0.1170	0.0930	0.0750	0.0630	0.0560	0.0500	0.0440	0.0390	0.0340	0.0300	0.0260	0.0220	0.0180
10,000	0.2150	0.1600	0.1200	0.0950	0.0760	0.0640	0.0570	0.0510	0.0450	0.0400	0.0350	0.0310	0.0270	0.0230	0.0190

<sup>1</sup> Amplitudes are for average overburden (computed from equation 5, p. 33). For outcrops divide amplitude by 10. For abnormal (deep or sand-gravel-loam) overburden multiply by 3.

TABLE 7.—Acceleration in terms of  $g^1$

Displacement, Inch	Frequency, <sup>2</sup> cycles per second						
	2	4	6	8	10	15	20
0.24	0.1	0.38	0.86	1.5	2.4	5.4	9.6
.22	.09	.35	.79	1.4	2.2	5.0	8.8
.20	.080	.32	.72	1.3	2.0	4.5	8.0
.18	.072	.29	.65	1.2	1.8	4.1	7.2
.16	.064	.26	.58	1.0	1.6	3.6	6.4
.14	.056	.22	.50	.90	1.4	3.2	5.6
.12	.048	.19	0.43	.77	1.2	2.7	4.8
.10	.040	.16	.36	.64	1.0	2.2	4.0
.08	.032	.13	.29	0.51	.8	1.8	3.2
.06	.024	.10	.22	.38	.6	1.3	2.4
.04	.016	.06	.14	.26	1.4	.9	1.6
.02	.008	.03	.07	.13	.2	.4	.8
.01	.004	.016	.036	.064	.1	.2	.4
.008	.0032	.013	.029	.051	.08	.2	.3
.006	.0024	.010	.022	.038	.06	.1	.2
.004	.0016	.006	.014	.026	.04	.09	.2
.002	.0008	.003	.007	.013	.02	.04	.08
.001	.0004	.0016	.0036	.006	.01	.02	.04
.0008	.0003	.0013	.0029	.005	.008	.02	.03
.0006	.0002	.0010	.0022	.004	.006	.01	.02
.0004	.0002	.0006	.0014	.0026	.004	.01	.016
.0002	.0001	.0003	.0007	.0013	.002	.004	.008
.0001	.0000	.0002	.0004	.0006	.001	.002	.004

<sup>1</sup> Computed from equation 11, p. 63.  
<sup>2</sup> Abnormal overburden, 4 to 10 cycles; average overburden, 10 to 20 cycles; outcrop, 20 to 80 cycles.  
 Representative frequencies: Average overburden, 15 cycles; abnormal overburden, 5 cycles; residential structures, 10 cycles.

On the other hand, an 8,000-pound shot at a distance of 500 feet would be dangerous because from table 6 the amplitude would be 0.14 inch. Table 7 shows that 0.14 inch at 10 cycles produces an acceleration of 1.4  $g$ , which is in the damage region.

As a whole, the regions in table 7 are conservative for quarry shots; that is, a combination that results in an acceleration in the damage region may not produce damage. If the combination of weight, distance, and overburden indicates an acceleration in the safe region, however, no damage will occur from ground vibration. The region marked "caution" represents combinations approaching dangerous proportions.

MISCELLANEOUS INVESTIGATIONS

DELAY BLASTING

Several tests were made to ascertain the feasibility of using delay blasting to reduce ground vibrations. Delay blasting is the process of firing a quantity of explosive in two or more sections with an

interval of time between sections. Although the idea of using delay blasting to reduce ground vibrations is not new,<sup>12</sup> usually its application has been restricted to geophysical prospecting. In this field interconnected geophones or pick-ups are spaced so as to introduce a time delay between the responses of individual units. Thus, the undesirable wave is not recorded but still exists in the ground.

By application of delay blasting to quarry practice it was hoped that the several sections of a delay shot could be timed so that the vibrational effect of one would be out of phase and counteract that of the preceding section and thus reduce vibrational movement.

The problem of minimizing ground vibrations consists of decreasing the vibration at a given point or over a given area. A decrease is desirable only insofar as it represents a true decrease in the vibration

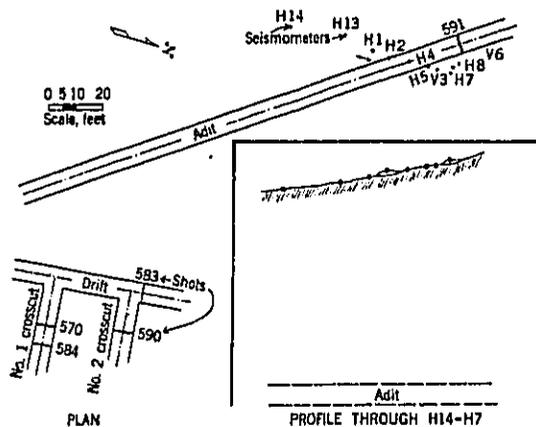


FIGURE 35.—Mount Weather testing well.

of the ground. The investigation utilized commercial blasting operations as well as experimental shooting so that the results would have practical significance.

As electric delay-blasting caps are in common use, the first comprehensive tests were made with them. Because the amount of delay in the caps cannot be controlled by the blaster, the main objective was to determine how the wave trains from different delays combined under actual blasting conditions, so that shooting of this type could be dealt with intelligently in the problem of damage from quarry and mine blasting.

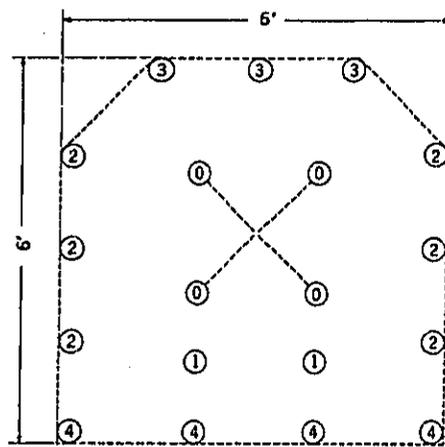
#### TESTS USING DELAY-BLASTING CAPS

This investigation was conducted at the Bureau of Mines Experimental adit, Mount Weather, Va. The adit, which is being driven through metamorphosed basalt, is indicated in figure 35 and described in detail in other Bureau publications (32).

<sup>12</sup> Taylor, H. G., Method of Recording Seismic Waves: U. S. Patent 1,790,308, Apr. 7, 1931.

The customary drilling and shooting methods were followed in the mine, except that one of the five shots was a test shot for another investigation (21). This test (shot 591) represents controlled shooting, in which the location of holes, amount of charge, and other factors were known to a high degree of accuracy. Figures 36 and 37 show the arrangement of holes for a regular shot and for the test shot.

The recording was done on the surface of the ground with 10 seismometers. Three of them (V6, H7, H8) were oriented along three mutually perpendicular axes; the other seven were arranged to measure horizontal movement along a straight line extending from



Holes 8 feet deep—delay number shown in hole  
 FIGURE 36.—Hole arrangement for regular shot (drift).

a point above the adit heading. The instruments were placed on rock "outcrops," which were probably large boulders or fragments rather than outcrops of rock in place.

The shots were loaded as shown in table 8. Each blast was detonated in five sections—one instantaneous and four delays.

TABLE 8.—Delay shots

Test No.	Location <sup>1</sup>	Dis- tance, feet	Delay-charge weight, pounds					Total
			0	1	2	3	4	
570.....	No. 1 crosscut.....	181	14.2	4.3	11.7	7.8	13.0	51
581.....	Drift.....	152	22.7	5.7	17.0	10.0	19.0	76
584.....	No. 1 crosscut.....	187	19.0	7.1	19.0	10.0	22.7	80
590.....	No. 2 crosscut.....	164	19.0	7.1	19.0	10.0	22.7	80
591.....	Adit.....	85	7.1	6.4	6.4	7.1	7.1	34

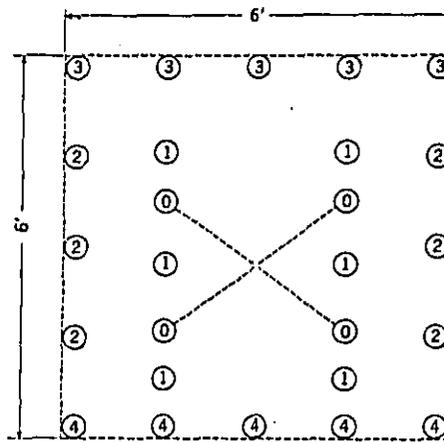
<sup>1</sup> See fig. 35.

<sup>2</sup> Straight-line distance to seismometer H14.

The total elapsed time for detonation of all sections was 3 seconds or more. After the instantaneous or zero-delay section detonated, 1 second elapsed before the No. 1 delay group detonated.

Inspection of the records showed that the number of individual wave trains exceeded the number of nominal delays in each round. For example, test 590 consisted of five delay sections (zero through No. 4), yet the record indicates six distinct wave trains. Furthermore, as the time intervals between sections were so great, the wave trains from separate groups could rarely, if ever, combine to form an increased amplitude under the conditions of the tests. In fact, the problem was not how the vibrations combined but rather how they separated to form more wave trains than delayed shots.

It was also noted that more than five reports were heard from many of the rounds. Variation in the cap timing in one delay group was



Holes 4 feet deep—delay number shown in hole

FIGURE 37.—Hole arrangement for test shot (adjt).

suggested as the reason for more "impulses" than shots. For example, a group of No. 4 delay caps might have enough difference in timing to cause two or more impulses, although the caps supposedly were designed to detonate at the same instant. Accordingly, tests were made on the caps to ascertain the amount of variation within each group.

#### TIMING OF ELECTRIC DELAY-BLASTING CAPS

Three groups of delay caps were fired separately by batteries. Each group consisted of five delay caps (zero through No. 4) selected at random and connected in series. Time of detonation was obtained by severing a small copper wire tied around the active end of each cap. Each detonation opened a resistance in a group of parallel resistances, thus changing the total current in the timing circuit. The timing-circuit current was recorded on a Duddell-type oscillograph.

Time could be read within about  $\pm 0.003$  second. The differences in time are given in table 9.

TABLE 9.—Time differences between zero delay and other delays, seconds

Test No.	No. 1 delay	No. 2 delay	No. 3 delay	No. 4 delay
585.....	1.602	2.181	2.700	3.140
586.....	1.636	2.218	2.733	3.321
587.....	1.633	2.300	2.749	3.290

From table 9 it is evident that the spread in cap timing of one group may be an appreciable percentage of the interval between groups. For example, the variation in the No. 4 delay caps for the first two tests is 3.321–3.140, or 0.181 second. The variation is about 35 percent of the 0.511-second difference between Nos. 3 and 4 caps of the last test.

DISPLACEMENT FROM DELAY CAP TESTS

A study of the amplitudes of vibration shows that the initial or zero-delay impulse is much greater than any of the following impulses. Table 10 gives the amplitudes recorded from the different rounds.

Table 11 shows the predominance of the zero-delay amplitude. For each seismometer it gives the ratio of maximum zero-delay amplitude to maximum amplitude of each wave train following the zero delay. All major wave trains were measured and numbered in sequence (1, 2, 3, etc.) following the zero delay. The ratios were averaged, and it was found that the amplitude of the zero delay was 11 times that of any other impulse.

TABLE 10.—Displacements

Test No.	Impulse No.	Seismometer No.—maximum single amplitude, inch									
		14	13	1	2	3	4	5	6	7	8
570.....	0	0.0035	0.002	0.004	0.0035	0.001	0.001	0.002	0.001	0.002	0.001
	1	.0005	.0003	.0004	.0003	.0001	.0002	.0001	.0002	.0002	.0001
	2	.0005	.0003	.0001	.0003	.0000	.0002	.0002	.0002	.0002	.0002
	3	.0003	.0001	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001
	4	.0003	.0002	.0003	.0002	.0001	.0001	.0001	.0002	.0002	.0001
583.....	0	0.0008	0.004	0.003	0.001	0.001	0.004	0.002	0.002	0.002	0.002
	1	.0005	.0002	.0003	.0002	.0003	.0003	.0001	.0002	.0002	.0002
	2	.0007	.0004	.0005	.0004	.0005	.0006	.0002	.0003	.0003	.0003
	3	.0007	.0003	.0003	.0003	.0003	.0004	.0002	.0003	.0003	.0004
	4	.0003	.0003	.0004	.0004	.0003	.0003	.0002	.0003	.0003	.0004
584.....	0	0.0004	0.004	0.004	0.0005	0.005	0.006	(?)	0.005	0.004	0.003
	1	.0006	.0004	.0003	.0004	.0007	.0008	(?)	.0004	.0004	.0003
	2	.0004	.0002	.0003	.0002	.0003	.0003	(?)	.0003	.0002	.0001
	3	.0008	.0003	.0003	.0004	.0007	.0008	(?)	.0004	.0004	.0003
	4	.0004	.0004	.0004	.0004	.0005	.0006	(?)	.0005	.0004	.0003
590.....	0	0.0001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	1	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
	2	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
	3	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
	4	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
591.....	0	0.0007	0.001	0.001	0.003	0.005	0.004	0.002	0.004	0.003	0.002
	1	.0002	.001	.001	.0025	.007	.0025	.005	.0025	.002	.002
	2	.0004	.0007	.0004	.0005	.002	.0002	.0001	.0001	.0001	.0001
	3	.0004	.0007	.0004	.0005	.002	.0002	.0001	.0001	.0001	.0001
	4	.0004	.0007	.0004	.0005	.002	.0002	.0001	.0001	.0001	.0001

1 Not readable.

TABLE 11.—*Ratios*

Test No.	Impulse	Seismometer No.—maximum single amplitude, ratio									
		14	13	1	2	3	4	5	6	7	8
570.....	0:1	7	7	10	12	10	5	20	5	10	10
	0:2	6	7	10	12	( <sup>1</sup> )	5	10	5	10	5
	0:3	12	20	20	25	( <sup>1</sup> )	10	20	10	20	10
	0:4	12	10	13	18	10	10	20	5	10	10
	0:5	7	10	20	12	10	5	10	10	10	10
583.....	0:6	4	5	13	35	10	2	10	5	10	5
	0:1	6	30	15	32	8	17	20	12	12	( <sup>1</sup> )
	0:2	4	15	9	16	5	6	10	12	8	( <sup>1</sup> )
	0:3	4	12	9	9	8	13	10	8	8	( <sup>1</sup> )
	0:4	5	20	11	16	8	17	( <sup>1</sup> )	8	8	( <sup>1</sup> )
	0:5	8	15	11	11	5	8	( <sup>1</sup> )	5	5	( <sup>1</sup> )
	0:6	5	16	9	16	4	6	( <sup>1</sup> )	6	6	( <sup>1</sup> )
	0:7	5	30	9	32	8	17	( <sup>1</sup> )	8	8	( <sup>1</sup> )
584.....	0:8	4	12	9	16	4	6	( <sup>1</sup> )	6	6	( <sup>1</sup> )
	0:1	( <sup>1</sup> )	5	9	( <sup>1</sup> )	8	7	10	4	4	10
	0:2	( <sup>1</sup> )	10	7	( <sup>1</sup> )	7	7	5	10	10	10
	0:3	( <sup>1</sup> )	5	14	( <sup>1</sup> )	15	20	( <sup>1</sup> )	5	20	20
	0:4	( <sup>1</sup> )	1	23	( <sup>1</sup> )	5	7	10	4	10	10
590.....	0:5	( <sup>1</sup> )	1	14	( <sup>1</sup> )	5	5	5	4	4	7
	0:1	10	10	35	22	5	13	5	5	13	7
	0:2	13	40	( <sup>1</sup> )	45	7	13	20	13	13	10
	0:3	8	8	7	7	4	7	10	2	8	5
	0:4	8	40	17	15	4	10	20	5	13	10
591.....	0:5	5	40	35	15	4	10	10	5	13	10
	0:1	7	6	8	5	10	10	50	5	5	10
	0:2	7	6	8	5	10	10	50	5	5	10
	0:3	7	6	9	5	10	10	50	5	5	10
	0:4	5	6	6	5	4	3	25	4	4	5
	0:5	5	6	4	5	4	3	25	4	4	4
	0:6	3	4	4	5	3	2	25	2	4	3
0:7	10	40	17	12	10	10	20	5	10	10	

<sup>1</sup> Amplitude too small for determination of ratio.  
<sup>2</sup> Not readable.

Table 8 shows that in every round at least one delay shot was comparable in weight of explosive charge to the zero-delay shot, therefore the high initial amplitude cannot be explained by weight of explosive alone. It is important to observe, however, that all of the explosive in any one delay shot does not necessarily detonate at the same instant because of irregularities in cap timing. Thus, the total weight of explosive of any delay shot does not necessarily detonate at the same instant but at two or more discrete intervals. If the charge for any delay group detonates at more than one interval, it is obvious that the displacement at any one interval cannot equal that of the full charge if detonated simultaneously; that is, the charges listed in table 8 may produce less vibrational effect than normal for the weights listed.

The decrease in vibration from this cause would be expected to predominate in the late delays insofar as the larger irregularities in time were found in these groups. Therefore, delay No. 1 probably will approach full effectiveness. Inspection of table 8, however, shows that in general the No. 1 delay shot (reliever) is a relatively small charge and even if fully effective (detonating at the same instant) could not be expected to produce the amplitude observed for the initial or zero-delay vibration.

Another factor that relates to the intensity of vibration is the amount of burden on the holes, or the "lightness" of the shot. Amplitude of ground vibration decreases with decrease in the burden carried by the hole (44). The zero-delay shot (cut holes), in addition

to being a heavy charge, is the "tightest" of the round, hence it will produce more vibration under the same conditions.

Table 12 lists the explosives for each shot. For all drift shots a higher-grade dynamite was used for the zero and No. 1 delays.

TABLE 12.—Dynamite<sup>1</sup> used in delay shots

Test No.	Delay, No.				
	0	1	2	3	4
570.....	A	A	C	C	C
583.....	A	A	B	B	B
584.....	A	A	B	B	B
590.....	A	A	B	B	B
591.....	A	A	A	A	A

<sup>1</sup> A=60-percent ammonia gelatin (0.71 pound per stick).  
 B=60-percent ammonia gelatin (0.71 pound per stick).  
 C=40-percent nitrostarch (0.63 pound per stick).

The higher-grade explosive may have physical characteristics associated with greater vibrational effect, which also might increase the amplitude of the zero and No. 1 delays.

These factors—cap timing, burden, and explosive characteristics—probably explain the relatively large initial or zero-delay amplitudes observed on the seismic records.

SUMMARY

1. The wave trains from the separate delay groups of a round fired with electric delay caps do not combine or overlap to increase the amplitude.
2. Each delay group of a round produces at least one vibrational impulse or wave train, and groups of greater delay generally produce two or more wave trains.
3. The additional wave trains are caused by irregularities in the cap timing.
4. The zero-delay explosive produces by far the largest seismic disturbance. The amplitude averages 11 times that of any later delay, even though the later delay may contain an equal weight of explosive.
5. The comparatively large zero-delay amplitude can be accounted for by three factors: (1) Cap timing, (2) burden on the shot, and (3) physical characteristics of the explosive.
6. The vibrations from each round lasted 3 seconds or longer, and the interval between the zero-delay and the first impulse of the No. 1 delay was never less than 1 second.

EXPERIMENTAL DELAY SHOTS

As electric blasting caps are manufactured so as to have comparatively long intervals of time between delay groups and are not consistent within a group, some other method of delay must be used if controlled or synthetic vibration is to be produced.

Instantaneous caps can be used and the delay produced by external means. For experimental work it proved convenient to take advantage of the current-time characteristics of electric caps. This charac-

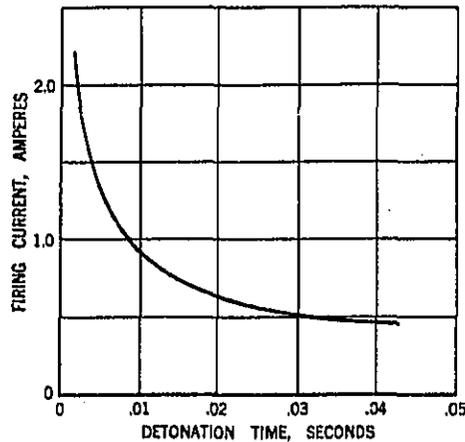


FIGURE 38.—Typical current-time characteristic of electric blasting caps.

teristic is illustrated in figure 38, which indicates that the detonation of a cap may be delayed by decreasing the number of amperes of current passing through it. Figure 39 shows a circuit that permits con-

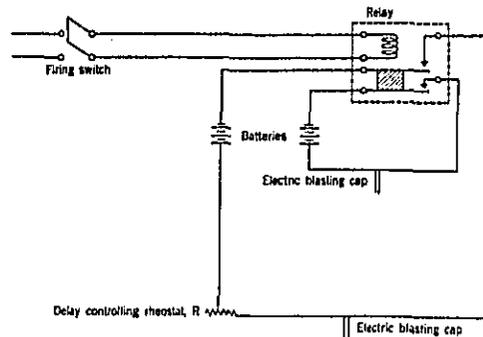


FIGURE 39.—Delay firing circuit for two caps.

trol of this type. The current and therefore the delay in detonation are varied by adjusting the rheostat. This method is restricted to delays of about 0.01 to 0.03 second and can be used to advantage only with caps in which the time characteristics have been closely duplicated.

With this delay method, small charges ( $\frac{1}{2}$  pound) were fired about 30 feet from the recording station, which comprised three mutually perpendicular seismometers.

First, a single shot was fired and the vibration recorded. The frequency of one horizontal component was fairly constant, as shown by test 494, figure 40, *A*. The firing apparatus was then adjusted so as to have a time delay equal to a half period of the vibration.

Two charges, each equal to the charge of test 494, were then fired at the same spot with the time delay between them. The resulting vibration record is shown in figure 40, *B*, test 497. Although the vibration (along the component being considered) is not eliminated, a substantial reduction is apparent (fig. 40, *C*), especially when it is

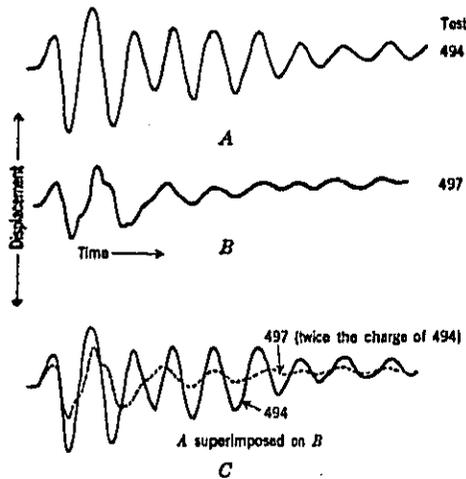


FIGURE 40.—Time-delay shots.

remembered that the weight of shot 497 was twice that of shot 494. If the frequency is not fairly constant, and often it is not, it is impossible to achieve substantial reduction in vibration by this simple method. It is possible, however, to cancel or eliminate a particular "peak" of the vibration, as illustrated in figure 41. This test differs from the previous one in that the delay was obtained not by employing delayed detonation but by spacing the charges so as to take advantage of the seismic travel times through the earth. In test 486 the charge was  $\frac{1}{2}$  pound of dynamite 50 feet from the recording point, whereas in test 488 it was  $\frac{1}{2}$  pound 30 feet from the recording point and diametrically opposed to test 486. The larger weight of explosive in test 486 was necessary to compensate for the greater distance. In test 490, also shown in figure 41, the shots of tests 486 and 488 were fired simultaneously. The result shows complete elimination of the first large peak, and the displacement very nearly equals

the algebraic sum of the individual displacements, as can be seen by comparing the sum (dotted curve, fig. 41) with the curve of test 490. Introducing the delay through proper spacing required considerable

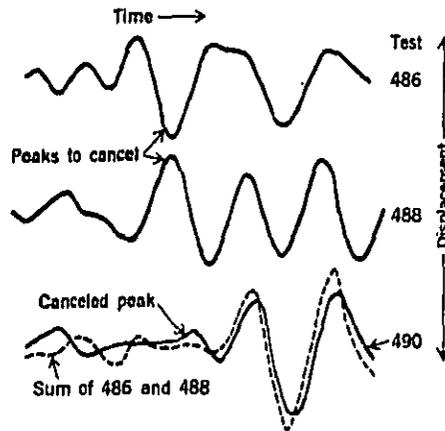


FIGURE 41.—Space-delay shots.

manipulation of both charge and distance, hence this method is impractical for commercial work.

#### DELAY SHOTS IN QUARRY BLASTING

As the experimental delay shots showed some promise of reducing vibration, a number of quarry blasts were shot using delayed firing.

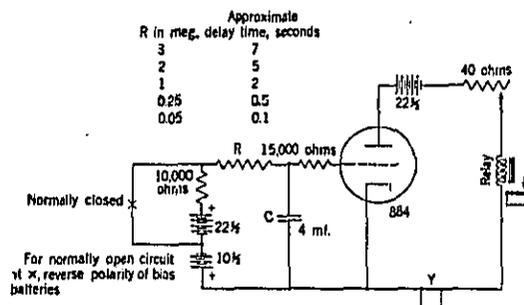


FIGURE 42.—Delay firing circuit.

The technique for producing the delay differed from that in the experimental shots because a delay range of about 0.02 to 15 seconds was necessary and the number of caps could not be restricted. Figure 42 gives the circuit devised for this control. The functioning of the circuit follows:

1. The initial or instantaneous detonation opens the circuit at *X* (by severing a wire).

2. Opening the circuit at *X* introduces a new voltage in the grid circuit. The new voltage is not applied immediately to the grid because of the drop in resistance *R* caused by the charge of current taken by condenser *C*.

The time required to charge *C* to about 63 percent of its full value is approximately  $C \times R$ , where *C* is the condenser capacity in farads and *R* the resistance in ohms. When the voltage across *C* has increased sufficiently, the gas in the tube breaks down and becomes a conductor, thus acting as a switch to close the relay circuit, which in turn fires the delayed shot.

The minimum delay is approximately the time necessary for closing the relay. The maximum possible time is well over 15 seconds.

The delayed section, when detonated, opens the circuit at *Y* through a "positive break" line and thus cuts off the plate current.

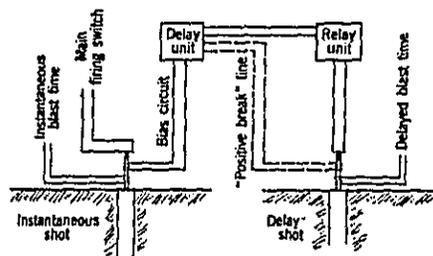


FIGURE 43.—Wiring circuit for delay shot.

Figure 43 indicates the method of inserting this delay and relay unit into the blasting circuit.

The first delayed-blast test was arranged at a limestone quarry. Seismometers were placed on overburden about 5,500 feet from the blasts. A record was made of a regular shot that contained 1,050 pounds of explosive. This record showed a fairly regular vibration in the direction of the shot with a predominant period of 0.13 second (frequency, about 8 cycles). This low frequency was characteristic of the region in which the observation was made. The delay unit was adjusted to give a delay of one-half the observed period, then inserted in the next comparable shot. Each of the two sections of this shot comprised 750 pounds of explosive.

The maximum amplitude resulting from the regular shot was 0.0005 inch, whereas that for the delay shot was 0.0002 inch. The difference in weight of explosive is not enough to account for this drop. Without further data, however, it is presumptuous to ascribe the decrease in amplitude to the delay alone. The result warranted further investigation.

Tests were made at the same quarry to determine the feasibility of decreasing the amplitude by adjusting the delay to correspond to the frequency observed at the quarry as differentiated from frequencies recorded 5,500 feet away. The records obtained in the quarry from regular shots, however, showed a very complex vibration with no

regular frequency, hence the adjustment of delay was more or less arbitrary. Furthermore, the tests showed that the effectiveness of the delay shot in reducing vibration under these conditions was nil.

To study the effectiveness of delayed shots where the delay interval was determined by a comparatively low frequency at the observation point, a number of delay shots were fired at a second quarry.

Observations were made on the first floor of a house and the ground outside. Measurements on floors above the first were influenced largely by the structure itself and were not used for these tests. The house was situated on a sand-gravel-loam deposit with characteristic low frequency of vibration.

Seven shots were recorded, and, of these, two were delay shots in which the delay was based upon the predominant frequency at the

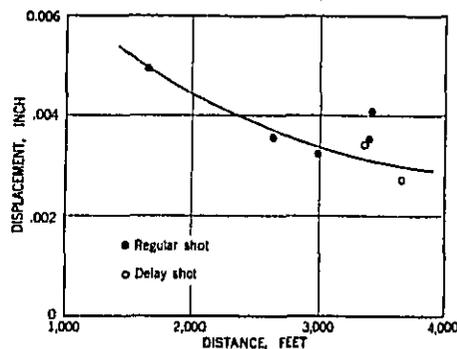


FIGURE 44.—Comparison of delay and regular shots, table 13.

house. Table 13 gives the characteristics of the shots and the displacements obtained. To compare the shots on an amplitude-distance graph, allowance must be made for the effect of weight of explosive. The correction of amplitude for weight is made by multiplying the observed amplitude by the square root of the ratio of a base weight of explosive to the true weight. The corrected displacements, listed in table 13, are plotted against distance in figure 44. Study of this graph shows that the two delay shots did not result in an appreciable decrease in amplitude.

TABLE 13.—Characteristics of shots

Shot No.	Total weight of explosive, pounds	Distance, feet	Resultant displacement, inch	Corrected displacement, inch
813.....	2,352	3,010	0.0035	0.0032
816.....	2,032	3,380	.0036	.0036
818.....	2,985	3,400	.0035	.004
817.....	2,081	3,620	.0035	.0035
815.....	2,980	1,675	.006	.005
810 <sup>1</sup> .....	4,450	3,300	.005	.0034
820 <sup>2</sup> .....	5,442	3,650	.0035	.0027

<sup>1</sup> Delay shot; first section, 2,316 pounds; second section, 2,143 pounds.

<sup>2</sup> Delay shot; first section, 2,220 pounds; second section, 1,203 pounds.

A similar series of tests was made in the same quarry, but the recording was done in another house situated on comparatively shallow overburden rather than on the sand-gravel-loam deposit. Again measurements were made on the ground and on the first floor for seven shots, two of which were delay shots.

The displacements are recorded in table 14. They were corrected for weight as in the preceding test. The corrected displacements are

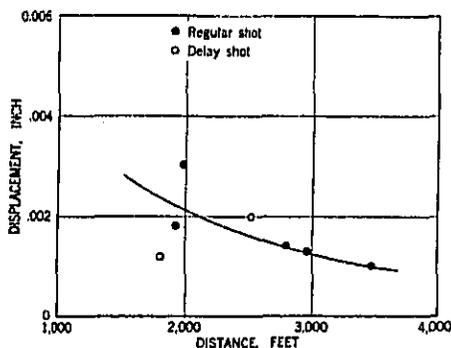


FIGURE 45.—Comparison of delay and regular shots, table 14.

plotted against the distance in figure 45. From this graph it is apparent that the delay shots did not appreciably decrease the amplitude.

TABLE 14.—Characteristics of shots

Shot No.	Total weight of explosive, pounds	Distance feet	Resultant displacement, inch	Corrected displacement, inch
827.....	2,376	2,975	0.0014	0.0013
828.....	2,202	2,810	0.0014	.0014
829.....	2,204	3,480	.001	.001
832.....	2,838	1,820	0.0014	.0013
833.....	1,315	1,925	0.0014	.0013
848.....	4,540	1,960	.004	.003
856.....	4,440	2,520	.003	.002

1 Delay shot; first section, 1,692 pounds; second section, 946 pounds.  
 2 Delay shot; first section, 2,250 pounds; second section, 2,250 pounds.

Delay-shot tests were also made in a mine. In these tests a delay-blasting machine was used. The time interval on a machine of this type is not readily adjustable, is quite restricted in range, and depends on the speed at which the machine is operated. Furthermore, if the machine has the firing circuits in parallel, misfires may occur in the delayed sections owing to a short across the circuit of the initial or zero-delay section. The short may be caused by the lead wires touching or grounding or by the cap wires contacting the cap shell. These faults are especially likely to occur in underground work but may be eliminated for the most part by insertion of a positive-break cap, which cuts off its own leg wires, at a point that

will not be disturbed by the blast. The results, given in a previous paper (44), also showed that the delay shooting had no advantage from a seismic standpoint.

#### SUMMARY

It is difficult to decrease vibration by delay blasting because often the vibration has no regular frequency, elimination of one component frequently does not appreciably affect the resultant amplitude, and the timing method must be quite flexible yet accurate.

It is concluded, therefore, that delay shooting of the type described in this paper is not practical for reducing vibration in commercial practice.

#### GENERAL SUMMARY

This study is based upon data collected from records of several hundred tests conducted at 28 stone quarries situated in 11 Southern and Eastern States, in a limestone mine, and in 20 residential structures of various types.

The tests covered the detonation of explosive charges in regular quarry practice ranging in weight from 1.5 to 42,000 pounds. Distances between shot point and seismometer stations ranged from 100 feet to 2 miles. Transmitting mediums through which the seismic waves were propagated ranged from granites through limestones, shales, and clays to unconsolidated sand and gravel beds.

Amplitudes of ground displacement as recorded ranged from 0.0001 to 0.06 inch and similar movements in structures from 0.0001 to 0.01 inch for quarry blasts and up to  $0.3 \pm$  inch for mechanical vibrations. Frequencies of the seismic waves ranged from 3 to 80 cycles per second and the duration of individual vibrations from 0.1 to 8 seconds.

#### CONCLUSIONS

1. Seismic vibrations emanating from quarry blasting, in which the size of the shots and the distances from shot to structure are not abnormal compared to customary quarry practice, produce no greater displacements of ground and residential structures than those produced by normal living activities within the structure or ordinary traffic conditions outside.
2. The magnitude of seismic displacement caused by quarry blasts can be predicted accurately enough for practical purposes if the weight of the explosive charge and the distance between shot point and structure are known.
3. Unconsolidated or abnormally thick overburden causes greater displacement at lower frequency than solid rock at equal distances and for equal weights of explosive charge.
4. Ground and structural displacements ranged from 0.0001 to 0.06 inch for quarry shots ranging in weight of explosive charge from 4 pounds at a distance of 185 feet to 15,400 pounds at 600 feet.
5. Buildings of one, two, or three stories can be vibrated in definite modes; their resonant frequencies and damping can be evaluated; hence their susceptibility to vibration can be determined.
6. Vibration of a residential structure at resonance does not in itself cause damage because of the restraining effect of damping inherent in the building.

7. Within the range of the tests made in this investigation, an acceleration equal to gravity ( $g=32.2$  ft. per sec.<sup>2</sup>) is a practical index of damage.

8. Customary vibrational disturbances from quarry blasting result in displacements and frequencies that represent accelerations of about 0.01 of gravity.

9. In tests carried to the damage point, damage occurred only when the acceleration nearly equaled or exceeded gravity.

10. The seismic vibration necessary to damage residential structures of the type tested in this study is much greater than that from ordinary quarry blasting.

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