LABORATORY TESTING OF PREDICTIVE LAND-USE MODELS: Some Comparisons

U.S. DEPARTMENT OF TRANSPORTATION
Washington, D.C. 20590
October 1976

U-96-01
II-A-264

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
DOT P 5010.		

4. Title and Subtitle	5. Report Date
Laboratory Testing of Predictive Land Use Models: Some Comparisons	October 1976
	6. Performing Organization Code
	8. Performing Organization Report No.
7. Author(s): Stephen H. Putman	
9. Performing Organization Name and Address: Department of City and Regional Planning, University of Pennsylvania, Philadelphia, Pa. 19174	10. Work Unit No. (TRAIS)
	11. Contract or Grant No.: NSF-GI-38978
12. Sponsoring Agency Name and Address	13. Type of Report and Period Covered: Final Report
	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The purpose of the research described here was to compare and evaluate operational land use forecasting models, and to suggest a procedure by which such comparisons and evaluations may be done for such models in the future. A thorough review of available operational models suggested two major types of models to be tested: the EMPIRIC, and the LOWRY derivative models.

The method of testing involved two parts. First, the parameters of each model were estimated for a common data base. Second, each model was used to make forecasts of the impact of different "policy" inputs, and the responses of each model to these inputs were compared.

The results of the parameter estimating suggest that either model can be mode to fit a base period data-set rather well. The lack of any explicit macro-behavioral structure to the EMPIRIC model makes interpretation of its parameters somewhat difficult. DRAM, the LOWRY derivative model which was tested, has a rather clear macro-behavioral structure and uielded parameters consistent with the theory underlying that structure. With regard to the policy tests, EMPIRIC was found to be very insensitive to changes in inputs. DRAM, by contrast, was properly sensitive to all the various policies tested. All in all, the testing of different models on a common data-set was a good method for comparison and evaluation.

17. Key Words	18. Distribution Statement
Land Use Models Urban Transportation Planning Urban Land Use Planning	

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price

Form DOT F 1700.7 (8-72) Reproduction of completed page authorized

LABORATORY TESTING OF PREDICTIVE LAND-USE MODELS: Some Comparisons

Report of Results From
NSF-GI-38978

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U.S. DEPARTMENT OF TRANSPORTATION
Office of Transportation Systems Analysis and Information
October 1976

Preface and Acknowledgement

During the past twenty years millions of dollars have been spent on design, development, and application of computer simulation models to urban problems. Despite the fact that at least one epitaph for urban simulation has been delivered, work continues to be done on this topic, which represents perhaps the only hope for comprehensive analysis capability for urban systems.

Due to the circumstances under which these models were developed, for the most part by private consulting firms, it has never been possible to make any true comparison of the many models which were attempted. Beginning in the late 1960's Professor Britton Harris and Professor Stephen H. Putman have collected, at the University of Pennsylvania, more than two dozen of these simulation models and their data sets. Due in part to this unique resource the National Science Foundation initiated the grant (NSF-GI-38978) which resulted in this report. The results reported herein are based on tests, utilizing actual urban data bases, of representatives of the two most frequently used forms of urban simulation model.

During the first half of the work under this grant the effort was directed jointly by Professors Harris and Putman. The final portion of the project and the preparation of this report were largely under Professor Putman's direction.

An external advisory board to the project provided valuable advice and especially helpful reviews of the draft of this report. The members of this board were:

Daniel Brand
William Garrison
David Gendell
Robert Goldman
William Goldner
Walter Hansen
John Hodge
Gregory Ingram
Philip Patterson
Wilbur Steger

Thanks are owed also to Professors David E. Boyce and Tony E. Smith of the Department of Regional Science, who provided useful comments and advice.

Many graduate students were involved with this project at one time or another. The following were responsible for significant components of the work:

Frederick Ducca
Charles Schramek
Ahmet Acar
Erhan Gencer
Rob Mathie
Charles Sawyer

Programming tasks were accomplished by:

Peter Kuner
Josefa Romanach
David Sanford

Secretarial work was ably provided by:

Pamela Riley
Margarita Gesualdo
Viriyane Norodom

Last, but far from least, we acknowledge the patient and helpful guidance of our NSF project monitors. The project was initiated with the help of Dr. Robert Goldman. After Dr. Goldman left NSF, Dr. David Seidman provided able and understanding guidance to the completion of the work.

CHAPTER ONE: INTRODUCTION AND SUMMARY

Introduction

The purposes of the research described here were to first review the status, and second develop a framework for comparison and testing of urban simulation models.

The development history of these models in the past twenty years has been cyclical. The initial model attempts began in the early 1960's and built to a crest of activity over a five year period. There followed, in the late 1960's and early 1970's a decline in modelling efforts in the wake of the many disappointments of the early work. Since the early 1970's there seems to have been a slow resurgence of model use and development.

Real progress in urban modelling can only be accomplished by a continuing process of model hypothesis and development, and subsequent model application and evaluation. Concentration on any one aspect of this process to the exclusion of the others probably causes no real harm, but may be an inefficient use of resources. Theory construction and statistical inference can never wholly substitute for empirical research. When this is attempted one is soon confronted by increasingly complex theoretical structures which simply cannot be supported by existing empirical foundations. Thus periodical evaluations and winnowing of previous results is a necessary part of the model development process.

Review and Selection of Models

There have been a number of good review articles published in past years.[1]

[1] Batty, M. (1972) "Recent Development in Land Use Modelling: A Review of British Research", Urban Studies, Vol. 9, No. 2, pp. 151-177.

Brown, H. J. et. al. (1972) Empirical Models of Urban Land Use New York: Columbia University Press.

Some further review work was done for this project and has been published elsewhere.[1] At the conclusion of the review it was decided that only the principal location algorithm(s) of the models would be tested. Thus more and less elaborate post-processing procedures, often found attached to these models, would be removed, allowing explicit evaluation of the model's basic construct.

The models reviewed, virtually all those for which any published descriptions were available, were classified into four broad groups.

A. The Lowry derivative models - this is now a large group of models[2] based on a straightforward set of relationships between place-of-work, place-of-residence, and in some cases, shopping-place. Most of these models deal with both residential location and non-basic or population-serving types of employment. All these models require an exogenously provided set of basic employment location estimates.

B. The EMPIRIC models - this is a somewhat smaller group of many applications of the same model.[3] The model is a set of linear difference equations with no explicit theoretical structure. The model applications involve statistical analyses of an urban data base, with the specific variables used in each application being determined as a consequence of their results. The models include both residential location and the location of all types of employment.

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[1] Putman, S.H. (1975) "Urban Land Use and Transportation Models: A State-of-the-Art Summary" Transportation Research, Vol. 9, No. 2, pp. 187-202.

[2] Goldner, W. (1971) "The Lowry Model Heritage" Journal of the American Institute of Planners, Vol. XXXVII, No. 2

[3] Hill, D. M (1965) "A Growth Allocation Model for the Boston Region" Journal of the American Institute of Planners, Vol. 1, pp. 278-287.

C. The research models - a small assortment of models with potential for application at some future time, but currently in the development or pilot application stage. Examples of these are the revised Herbert-Stevens model, the National Bureau of Economic Research (NBER) model, and the Birch model.[1]

D. A miscellany of other models - a group of models proposed but not implemented, implemented but not successfully, implemented but too complex or tailor-made to a particular circumstance to allow application elsewhere, and others simply not worth pursuing further.

Having grouped the models this way, it is quite clear that models from the last two groups were not appropriate for further investigation. The most useful comparison of models then appeared to be a comparison between an application of EMPIRIC and an application of a Lowry derivative model.

The EMPIRIC model has been applied in a dozen or more major cities of the U.S. Any one of these applications would have been suitable for our comparison purposes.

There have been almost as many applications of Lowry derivative models in major U.S. cities. Of these, the most frequently applied model has been the Projective Land Use Model (PLUM) in any of its several versions. Consequently the initial intent of this project was to compare a version of PLUM to one of the EMPIRIC applications.

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[1] Wheaton, W. Jr. (1974) "Linear Programming and Locational Equilibrium" Journal of Urban Economics, Vol. 1, pp. 278-287.

Ingram, G. et. al. (1972) The NBER Urban Simulation Model New York: Columbia University Press.

Birch, D. et. al. (1973) "The New Haven Laboratory: A Testbed for Planning" Report to the U.S. Dept. of Housing and Urban Development.

Comparison of Models: Parameter Estimation

The first comparison of the two models was to be with respect to the difficulties and relative success in estimating their parameters. This aspect of the project led to one of its major research findings. After careful review of all Lowry derivative applications in the U.S. it was discovered that in all but one case[1] the model parameters had not been properly estimated. A careful investigation was subsequently made into British modelling practice, where this problem had been identified and largely resolved. This led to a reformulation of the version of PLUM originally scheduled to be used in the project, and the subsequent development of a new form of the model, called Disaggregated Residential Allocation Model (DRAM). DRAM was used throughout the remainder of the project; it is further described in Chapter 2 and Appendix 1 of this report.

The estimation of parameters for EMPIRIC was more straightforward and was accomplished using the same procedures as had been used in its various applications. EMPIRIC's parameters were reestimated for Boston, Minneapolis-St. Paul, and Washington, D.C. DRAM's parameters were estimated for San Francisco and Minneapolis-St. Paul. Work was also done to secure other data sets for further parameter estimations in future project efforts. In most cases EMPIRIC yielded a slightly better fit to the base year data than did DRAM. This was accomplished by use of a much more extensive set of independent variables, but in the absence of any behavioral structure to the model. DRAM, with slightly lower base year data fits, but with much reduced data input requirements, is likely to produce better long term forecasts than EMPIRIC. More detailed results of these parameter estimations are presented in Chapter 2 of this report.

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[1] The Voorhees Urban Systems Model (USM) application in the Dallas-Fort Worth, Texas region.

Comparison of Models: Sensitivity Tests

The parameters of both EMPIRIC and DRAM (plus an associated employment estimating model, EMPMOD) were thus estimated for the Minneapolis-St. Paul data set. Following this, the models' responses to both arbitrary changes in inputs as well as to simulated policy inputs were tested. Many of these tests were performed to test the models' responses to varying circumstances. Very important differences showed up in the models' performances in these tests. EMPIRIC showed no population response to changes in base year population or employment, and employment response only to base year employment changes. EMPIRIC showed some response to zone specific accessibility changes, but no response to regionwide changes. DRAM, in all these cases showed what appeared to be proper responses to all these changes in inputs. These results are presented in more detail in Chapter 3 of this report.

Conclusions

The principal conclusions of the research effort are enumerated here. All are described in more detail in the following chapters.

1. Both models require substantial data preparation prior to their use.
2. The parameters of either model can be adjusted to yield rather close statistical fits to observed data.
3. Based on these fits, both models appear to be capable of making forecasts of urban form in the absence of attempted policy manipulations. EMPIRIC may have a slight advantage over DRAM in this respect.
4. DRAM is clearly superior with respect to its response to changes in inputs. This suggests a clar advantage over EMPIRIC whenever policy tests are contemplated.
5. Testing different models on common data sets appears to be a powerful means for comparing models.

CHAPTER TWO: THE MODELS AND CALIBRATIONS

The Models Briefly Described

A brief description of the models used in the project is presented here, prior to discussing their calibration.

The EMPIRIC model is basically a set of linear equations. The variables are all expressed in terms of regional shares. For example, taking population by zone;

$$p_i = P_i / \sum_i P_i$$

where

p_i = the share of the region's population found in zone i

P_i = actual population of zone i

The dependent variables in EMPIRIC are expressed as changes in a zone's share of the variable from time t to time t+1. For example, again taking population.

$$\Delta p_i = p_{i,t+1} - p_{i,t}$$

Finally, the EMPIRIC equation structure is simultaneous, each dependent variable being a function of other dependent variables plus several predetermined variables (either lagged variables or exogenously determined variables). More specific discussion of the actual variables used will be found in the next section of this chapter.

These definitions comprise the full extent to which EMPIRIC has any structure. The variables to be included in each equation are not prespecified. Their selection is generated as the output of statistical analyses in each model application. It is precisely this lack of a substantive theoretical form which justifies the econometricians' contention that the model is not properly specified, and the urban modellers' contention that the model is non-behavioral.

The DRAM model is a sophisticated variation on the basic Lowry model theme. The hypotheses of the Lowry model assert that, given a spatial distribution of employment, and a description of zone-to-zone travel times (or costs) it is possible to estimate the location of the employees' residences. This location is taken to be a result of trip length probabilities, and in the more complex variants of the model, of residence zone characteristics. This may be written in equation form as,

$$P_i = \sum_j E_j \, p_{ij} \, A_i$$

where

P_i = population of zone i

E_j = employment in zone j

p_{ij} = the probability of a work trip as long as the travel time (or cost) between zones i and j

A_i = a measure of the residential attractiveness of zone i

The important differences between the Lowry variants result from different functional forms to generate p_{ij} from travel times (or costs) and different ways of measuring A_i. Further differences stem from structuring the model in static or dynamic form.

In DRAM,

$$p_{ij} = f\{D_{ij}^{\alpha} \; \exp \; \beta D_{ij}\}$$

where

D_{ij} = travel time between zone i and zone j

α, β = empirically derived parameters

and

$$A_i = f\{X_{1i}^{\delta 1}, \; X_{2i}^{\delta 2}, \; \ldots, \; X_{ni}^{\delta n}\}$$

where

$X_{ni}^{\delta n}$ = various measures of zonal characteristics including population composition and level of development

δn = empirically derived parameters

The process of finding numerical values for the parameters α, β, and δn is described later in this chapter and involves both equation fitting and, due to the explicit structure of the model, hypothesis testing.

All the Lowry variants require some external source of basic employment estimates. In past practice these sources have been quite varied, ranging from hand prepared estimates to rather complex models. In order to skip these complications in the present work, a straightforward multiple regression model, EMPMOD, was assembled for the purpose. The development of EMPMOD was incidental to the prime focus of this work. While the estimates produced are reasonably good (and are discussed later in this chapter) EMPMOD should be considered a means to the project's end.

The remainder of this chapter is devoted to a discussion of the models' parameter estimations and their implications.

Calibrating the Models

In the development of models of urban and regional systems the analyst is irrevocably trapped in the problems of drawing inferences from non-experimental statistics. It is not possible, say, to have two San Francisco Bay Areas on which to run controlled experiments. A direct consequence of this vexing situation is that we can never prove the ultimate correctness of any given model formulation as opposed to any other. We may eliminate many possible formulations on the grounds that they conflict with accepted theory and/or empirical results. Once we have narrowed down to a few likely candidates for further testing there really is no way to prove that one is better than another. We are, however, willing to assert that a model which achieves good fits to data is more worthy of further investigation than one which does not fit as well.

Considerable progress has been made in developing methods for finding "best-fit" parameters of urban simulation models. The process of finding a set of numerical parameters for a specific equation (or set of equations) which produce the best fit of those equations to a given data set has come to be called calibration of the equation(s) or model(s). In a particular example of a given data set and a given set of equations, any procedure for adjusting parameters to fit the equation to the data may properly be called a calibration procedure. The important questions here, given the data and equation(s), are first whether the procedure is computationally efficient and consistent with the theoretical structure (equations) of the model, and second, how to measure the goodness-of-fit of the model to the data.

It is not always possible to determine the best calibration procedure. It is often possible to eliminate some procedures as being clearly less adequate than others. One example of an inadequate procedure would be the practice of fitting parameters to one part of a model without taking into account their interactions with other parameters in the model. Another such example would be the practice of arbitrarily assigning parameter values without testing the consequences of such values. Neither of these procedures could produce a "proper" calibration of a model.

It is likewise not possible to specify a best fit criterion which is applicable under all circumstances. The coefficient of determination R^2, is often used as a measure of goodness of fit. Yet, this measure is, strictly speaking, inappropriate for the nonlinear models often encountered in urban simulation. Other more appropriate criteria, such as maximum likelihood estimates, are so little known to model practicioners as to be viewed with some trepidation. Criteria such as root mean square error, or standard error of estimate, do not provide a convenient basis for comparing one model to another in the absence of identical data sets.

The procedures and criteria used in this study are described along with the discussion of calibration results which follows.

Calibration of EMPIRIC

The EMPIRIC model was first described more than a decade ago, and has since seen application in more than a dozen U.S. cities. Peat, Marwick, Mitchell & Company (hereafter referred to as PMM) have been the principal proponents and practicioners of EMPIRIC. In past years they have generously supplied reports and data from these applications to the Principal Investigator of this study. Consequently there were detailed descriptions of previously estimated EMPIRIC models available for this study. These reports were available for the Atlanta, Boston, Denver, Puget Sound, Twin-Cities (Minneapolis-St. Paul), and Washington,D.C. metropolitan areas. In addition there were packages of computer programs and data sets available for Boston, Twin-Cities, and Washington. An idea of the sizes of these metropolitan areas as modelled may be obtained by reference to Table 1.

Reviewing each of these applications led to the conclusion that while many of the variables used were similar from one application to the next, (the equation structure was, of course, identical), the specific variables used were different in each application. The dependent variables were always expressed in terms of change in regional share. Population was always divided into four groups, by income, approximating quartiles. These groups are referred to as Lower Income, Lower Middle Income, Upper (or Higher) Middle Income, and Upper (or High) Income.

The five EMPIRIC equation sets were then examined for evidence of consistencies or inconsistencies from one model application to the next. In each application there were, typically, four or five population sectors and five or six employment sectors being forecasted. The precise sectoral definitions differ from one application to the next, but are generally similar.

As above, the population is usually defined as household income quartiles or groups approximating quartiles, while employment usually consists of a few basic and a few non-basic sectors. For each sector, the dependent variables are change in the zone's share of the region's total amount of the particular activity. The independent variables are of four types. First, there are lagged, or base year, values of the dependent variables and second, there are the other dependent variables. The third type of independent variable is the accessibility and/or land use variables of which there are usually several. Finally there are the public utility variables such as sewer and water availabilities.

Name of Region	Population	Employment	Year	Counties in Study Region
Atlanta, Ga.	1.0 million	605 thousand	1961	7
	1.4 million	n.a.	1970	
Boston, Mass.	3.4 million	n.a.	1960	n.a.
Denver, Colo.	0.9 million	388 thousand	1960	5
	1.2 million	533 thousand	1970	
Puget Sound, Wash.	1.7 million	610 thousand	1970	4
Twin-Cities, Minn.	1.5 million	610 thousand	1960	7
	1.9 million	850 thousand	1970	
Washington, D.C.	2.1 million	1146 thousand	1968	7

Table 1: Comparative Sizes of EMPIRIC Application Regions

The general procedure involved in applying the EMPIRIC model involves first, the preparation of a large file of raw (i.e. corrected, but unmodified) and constructed (i.e. combinations or modifications of raw) variables. A selection is then made of variables, generally those which have worked well in prior applications, for use in the preliminary regression analyses. The completion of the model calibration is then a matter of testing alternative variables until a best fit set of equations and parameters is obtained. EMPIRIC is, in a sense, very much an opportunistic model in that the final selection of variables to be used is largely based on the results obtained in the regression analyses. Those variables which produce the best fit being the ones used in the model. The regression fits obtained by this means are generally good, with coefficients of determination ranging upwards from 0.55, many of them being in the range of 0.70 to 0.90.

The measure of goodness of fit used in the EMPIRIC applications was the multiple coefficient of determination R^2. These results are tabulated for the various studies in Table 2. Note that there are two sets of results for most regions. These represent the R^2 from calibration or fitting the model to the data set, and the R^2 from reliability tests. The reliability tests consisted of using the fitted model to forecast the second data point (e.g. 1970) from the first (e.g. 1960) and then comparing the forecast to the actual data (e.g. estimated 1970 vs. actual 1970).

In Table 3 are shown the coefficients of the population variables used in the final versions of the EMPIRIC population equations for each region. A fair degree of consistency is found here, though there are some obvious discrepancies both in sign and magnitude of these coefficients. Note that the coefficients

Name of Region	Number of Zones	Time Period	Test Type	Lower Income	Lower Middle	Upper Middle	Upper Income
Atlanta	183	1961-70	Calib.	0.558	0.792	0.812	0.770
	290	1961-70	Reliab.	0.540	0.670	0.810	0.830
Boston	104	1950-60	Reliab.	0.990	0.950	0.915	0.946
	453	1950-60	Reliab.	0.951	0.906	0.793	0.826
Denver	183	1960-70	Calib.	0.647	0.841	0.855	0.839
			Reliab.	0.938	0.890	0.702	0.694
Puget Sound	244	1961-70	Calib.	0.573	0.719	0.900	0.850
			Reliab.	0.880	0.816	0.822	0.855
Twin-Cities	108	1960-70	Calib.	0.702	0.708	0.812	0.715
			Reliab.	0.919	0.940	0.880	0.827
Washington, D.C.	110	1960-68	Calib.	0.698	0.770	0.844	0.750
			Reliab.	0.947	0.917	0.877	0.886

Table 2: Fitting and Reliability Results - R^2 for Several EMPIRIC Applications for the Four Population Classes

Table 3: POPULATION COEFFICIENTS IN EMPIRIC MODELS

		Population by Income (Independent Variable)							
		Change in Share				Base Year Share			
Dependent Variable	Study Area	Lower	Lower Middle	Upper Middel	Upper	Low	Lower Middle	Upper Middle	Upper
Change in Share Low Income Population	Atlanta	-.119	+.558	-.367					
	Denver		+.129			-.392	+.337		
	Washington		+.229		-.281	-.199		+.258	
	Twin Cities		+.40		-.39	-.42	+.36		
	Puget Sound		+.352			-.314	+.294		
	Boston		+.637	-.295		+.133		-.109	
Change in Share Low-Middle Income Population	Atlanta	+.512		+.480					
	Denver	+.201		+.307			-.353	-.334	
	Washington	+.194		+.781			-.279	-.182	
	Twin Cities	+.28		+.45				+.10	
	Puget Sound			+.531			-.054*		
	Boston	+.53		+.337•			-.101		
Change in Share Upper-Middle Income Population	Atlanta		+.439		+.338				
	Denver		+.612		+.25			-.27	
	Washington		+.658		+.399				-.155
	Twin Cities	-.14	+.45		+.26	-.16			
	Puget Sound		+.434		+.43			-.219	+.113
	Boston	-.125	+.637		+.294			-.224	
Change in Share Upper Income Population	Atlanta		+.512						-.447
	Denver		+.685						-.481
	Washington	-.507		+.504					
	Twin Cities	-.42		+.83					
	Puget Sound			+.657				+.219	-.437
	Boston		-.282	+.603					-.278

*Base Year Share / Total Household

shown are those which were statistically significant, as those which were not significant are not published in the PMM reports.

An interesting pattern shows in Table 3. For each population class, the change in share of a region's total population found in each zone, moves with the change in share of the adjacent population class, viz; Lower Income moves with change in share of Lower Middle Income, Lower Middle Income moves with change in shares of Lower Income and Upper Middle Income, and so on. Further, for each population class, change in share moves in opposition to (i.e. the signs of the coefficients are negative) its own concentration in the base year and moves with (though the pattern is weaker) concentrations of the next higher income group. Stated in other words, changes in share by zone of each income group move 1) with changes in shares of the next higher and next lower income income group, and 2) away from concentrations of their own income group towards concentrations of the next higher income group.

The patterns found in these coefficients of the population variables are quite consistent with hypotheses regarding peoples desires for increased socio-economic status, as well as with hypotheses regarding peoples unwillingness to live among groups very different from themselves. The patterns of coefficients of other variables in the population equations as well as those of the variables in the employment equations do not exhibit a similar degree of uniformity, and consequently are not tabulated here, though the specific case of the Twin-Cities application is discussed in more detail below.

In the other portions of these EMPIRIC model equations the sense and sensibility of the variables used, and their coefficients is another matter. There are a number of instances of contraintuitive coefficient signs and many constructed variables whose real meaning is somewhat obscure. An harsh critic could assert that the equations derived all their correlations from the unavoidable implicit correlations between activities in urban areas. Thus from the causal point of view the model results could be called fortuitous and/or spurious. A more reasonable position would be that the equation sets depend, to a significant degree, upon these strictly associative relationships, but that they will probably produce reasonably good near term forecasts, taken all together. Another view of these equations is that they are the reduced form of structural equations (in the econometric sense) which are unknown. If this view is correct, as it well may be, the use of these equations for forecasting requires that both the structure and the parameters of the unknown structural equations remain constant over the forecast period. Problems arise, as will be discussed later in this report, when policy tests with this model are attempted by means of changing specific variables. In the absence of a known, or even of an assumed structural form, it is likely that changing variables in the reduced form equations will produce peculiar results. That this concern is justified will be amply demonstrated in the discussion of sensitivity tests of EMPIRIC in a later chapter of this report.

As part of this project the three EMPIRIC applications for which data were available were all run several times, to the end of becoming more familiar with their operation. Of these three, Boston, Washington, D.C., and the Twin-Cities, recalibration runs were made for the Boston and Twin-Cities data sets. For the Twin-Cities data set the equations presented in the PMM final report were rerun using both ordinary least squares (OLS) regression and two stage least squares (TSLS), regression.[1] The differences between the OLS and TSLS

[1]Peat, Marwick, Mitchell and Co. (1971). "Calibration and Application of an EMPIRIC Activities Allocation Model for the Twin-Cities Metropolitan Area", prepared for the Metropolitan Council, St. Paul, Minneasota.

calibration reruns were minor, as were all but one of the differences between the PMM calibration and these calibration reruns. The reason for the one larger difference is neither known nor important in the context of this project. The differences in coefficients were also minor in all cases . The variable definitions for this EMPIRIC application are shown in Table 4. The statistically significant coefficients of the equations for the TSLS calibration rerun are given in Table 5.

The great number of constructed variables used in the EMPIRIC equations make it rather difficult to interpret the results of the parameter estimations. There are few consistencies to be found in this parameter set. There are many peculiarities to be mused over. Why is change in a zone's share of population in the low income quartile positively related to change in local government and educational employment and negatively related to change in the product of highway accessibility to employment and used land area? Why is change in a zone's share of population in the upper middle income quartile not related to any employment or access variable? Why is change in a zone's share of population in the high income quartile positively related to the base year industrial employment as proportion of total employment in the zone; and not related to any other employment or access measure? More generally why aren't the EMPIRIC variables described as relative values rather than shares, thus avoiding the need to interpret what a zone's share of the percentage of something in the zone implies?

In the absence of an explicit theory or an attempt at structural equations, there can be few expectations regarding signs and magnitudes of coefficients. Consequently there is little point in discussing the EMPIRIC calibration results at length. Suffice it to say, the parameters of EMPIRIC model can be calibrated to yield relatively close fits to the data. The only consistency in the parameters

Table 4: VARIABLES DEFINITIONS - TWIN CITIES EMPIRIC

Note: Shares means regional share of variable X to be found in zone

Δ indicates "change-in-share" variables; all others are base year shares.

LIQ	=	Households in lowest income quartile
LMIQ	=	Households in lower-middle income quartile
UMIQ	=	Households in upper-middle income quartile
HIQ	=	Households in highest income quartile
MISC	=	Construction and other miscellaneous employment
MFGW	=	Manufacturing and wholesale employment
TCU	=	Transportation, communications, utilities employment
RET	=	Retail employment
SVCFIR	=	Service, finance, insurance, real estate employment
LGOVED	=	Local government and education employment
HAHU	=	Highway accessibility to households
TAHU	=	Transit accessibility to households
AHU	=	Composite (sum of highway and transit) accessibility to households
HAEMP	=	Highway accessibility to employment
AEMP	=	Composite accessibility to employment
SEWER	=	Percent of district "sewered"
NCA	=	Net commercial area
NIA	=	Net industrial area
NPA	=	Net public and semi-public area
USEDAC	=	Used area = NCA + NIA + NPA + net residential area
VACAC	=	Vacant or agricultural area
DEVAC	=	Developable area = USEDAC + VACAC
TOTAC	=	Total area of the district
TOTHU	=	Total housing units
TOTEMP	=	Total employment
NRA	=	Net residential area

Table 5: EMPIRIC EQUATIONS FOR TWIN CITIES - U. OF P. TWO STAGE LEAST SQUARES ESTIMATES

Note: Variables are 1960 share or (for Δ variables) change in share 1960-1970. R^2 for these equations are given in Table 6

$$\Delta \mathrm{LIQ} = 0.407\Delta\mathrm{LMIQ} - 0.377\Delta\mathrm{HIQ} + 0.106\Delta\mathrm{LGOVED} - 0.415\mathrm{LIQ} + 0.357\mathrm{LMIQ} - 0.890\Delta(\mathrm{HAEMP} * \mathrm{USEDAC}) + 0.269\Delta\mathrm{SEWER} + 0.060(\mathrm{SEWER} * \mathrm{VACAC}) + 0.112\,(\mathrm{TOTEMP}/\mathrm{TOTHU})$$

$$\Delta \mathrm{LMIQ} = 0.299\Delta\mathrm{LIQ} + 0.425\Delta\mathrm{UMIQ} + 0.092\mathrm{UMIQ} - 0.109(\mathrm{AEMP} * \mathrm{USEDAC}) + 0.300\Delta(\mathrm{HAEMP} * \mathrm{USEDAC})$$

$$\Delta \mathrm{UMIQ} = -0.144\Delta\mathrm{LIQ} + 0.415\Delta\mathrm{LMIQ} + 0.261\Delta\mathrm{HIQ} - 0.163\mathrm{LIQ} + 0.058\,(\mathrm{SEWER} * \mathrm{TOTAC}) + 0.104\,(\mathrm{UMIQ}/\mathrm{TOTHU})$$

$$\Delta \mathrm{HIQ} = -0.416\Delta\mathrm{LIQ} + 0.0\Delta\mathrm{LMIQ} + .830\Delta\mathrm{UMIQ} + .248\Delta\mathrm{SEWER} - .260(\mathrm{HIQ}/\mathrm{TOTHU}) + .274(\mathrm{INDUS}/\mathrm{TOTEMP})$$

$$\Delta \mathrm{MISC} = .44\Delta\mathrm{RET} + .20\Delta\mathrm{SVCFIR} - .026(\mathrm{TOTEMP}/\mathrm{TOTHU}) + .112(\Delta\mathrm{SEWER} * \mathrm{TOTAC}) - .256\mathrm{MISC} - .096\mathrm{SVCFIR} + .109(\mathrm{NIA} * \mathrm{VACAC}/(\mathrm{USEDAC} + \mathrm{VACAC}) + .094\mathrm{TAHU})$$

$$\Delta \mathrm{MFGW} = .013\Delta\mathrm{SVCFIR} + .190(\mathrm{SEWER} * \mathrm{TOTAC}) + .254\mathrm{SVCFIR} - .189\mathrm{MFGW} - .268\mathrm{NCA} - .531(\mathrm{USEDAC}/(\mathrm{USEDAC} + \mathrm{VACAC})) - .248\Delta\mathrm{HAHU} * \mathrm{USEDAC} + .52\mathrm{HAHU}$$

$$\Delta \mathrm{TCU} = .737\Delta\mathrm{RET} + .919\Delta\mathrm{SEWER} + .249\mathrm{NIA} * \mathrm{VACAC}/(\mathrm{USEDAC} + \mathrm{VACAC}) - .352\mathrm{MFGW} + .60\mathrm{USEDAC}/(\mathrm{USEDAC} + \mathrm{VACAC}) + .182\mathrm{TCU} - .53(\mathrm{TOTEMP})/(\mathrm{NIA} + \mathrm{NCA} + \mathrm{NPA}) + .31(\mathrm{TOTEMP}/\mathrm{TOTAC}) - .423(\mathrm{NCA} * \mathrm{VACAC}/(\mathrm{USEDAC} + \mathrm{VACAC}))$$

$$\Delta \mathrm{RET} = .473\mathrm{SVCFIR} + .518\Delta\mathrm{LMIQ} + .077\mathrm{NCA} * \mathrm{VACAC}/(\mathrm{USEDAC} + \mathrm{VACAC}) - .32\mathrm{RET} + .291\mathrm{AHU} * \mathrm{USEDAC}$$

$$\Delta \mathrm{SVCFIR} = .169\Delta\mathrm{UMIQ} + .202\mathrm{MFGW} + .344\mathrm{RET} - .154\mathrm{GOVED} - .228\mathrm{SVCFIR} + .236\mathrm{RET}$$

$$\Delta \mathrm{LGOVED} = .291\Delta\mathrm{LIQ} + .313\mathrm{TAHU} + .214\mathrm{NCA} - .539\mathrm{LGOVED}$$

Table 6: COMPARISON OF CALIBRATIONS OF EMPIRIC: TWIN-CITIES DATA

	1	2	3
Dependent Variable	PMM-R^2	UoP-TSLS-R^2	UoP-OLS-R^2
ΔLIQ	0.702	0.703	0.706
ΔLMIQ	0.708	0.714	0.720
ΔUMIQ	0.812	0.816	0.824
ΔHIQ	0.715	0.715	0.724
ΔMISC EMP	0.750	0.746	0.761
ΔMFG	0.718	0.708	0.714
ΔTRANSP	0.504	0.464	0.464
ΔRET	0.790	0.790	0.793
ΔSERV+FIRE	0.755	0.754	0.758
ΔLOGOV+ED	0.545	0.545	0.546

Column 1 - Resulting R^2 from PMM calibrations

Column 2 - Resulting R^2 from this project's recalibration using Two Stage Least Squares regression.

Column 3 - Resulting R^2 from this project's recalibration using Ordinary Least Squares regression.

Identical dependent and independent variables were used in all three calibrations.

from one application to the next appears in the population group-to-population group relationships. The parameters for other variables and other equations are catch as catch can, and raise questions as to the simultaneity alluded to in the general descriptions of the model which accompany each application. Overall, attempts to use these models for any but short term, no policy, forecasts should be viewed with considerable skepticism.

Calibration of DRAM

During the initial stages of this project the decision was taken to compare the EMPIRIC model to a package containing a version of IPLUM and a simple basic employment model. There was no intention of performing any model development work for this project. This was all well and good as the project proceeded on through its early stages. It was when work began on the calibration of IMPLUM, that trouble became apparent. In fact, the entire history of application of Lowry and "Lowry derivative" type models in the U.S. is fraught with tales of calibration difficulties. Further investigation yielded the unpleasantly interesting fact that in U.S. practice, with but one exception, no Lowry type model had ever been successfully calibrated (in a statistical sense). Partial calibrations, in some cases of the p_{ij} function, in others of a multivariate measure of A_i, had been accomplished, but no complete estimations of a model's parameters had been done. There had, however, been a number of successful calibrations of Lowry type models in British practice. Consequently an effort was undertaken to determine whether IPLUM could be calibrated by the procedures used in the British work.

The British calibrations draw upon a reformulation of Lowry type models according to the Wilson maximum-entropy approach.[1] This approach was also

[1] Wilson, A. (1970). Entropy in Urban and Regional Modelling, Pion Ltd., London.

used in the one U.S. exception mentioned above, the Voorhees U.S.M.[1] When so reformulated, the path to calibration of Lowry type models becomes quite clear, though it does require use of mathematical search techniques for non-linear equations, rather than the better known multiple regression techniques used in EMPIRIC. Consequently it became necessary to recast IPLUM in the entropy maximizing form. As a part of this effort several very desirable improvements to the model became not only possible, but, in a sense, inevitable. In particular, the population sector of the model, formerly considered as one homogenous group, was disaggregated into four sectors defined in terms of income. Further the need for many of the arbitrary correction factors used in the later portions of the IPLUM model was eliminated. This new formulation of the model eventually became sufficiently different from its progenitor to warrant a new name - Disaggregated Residential Allocation Model (DRAM).

The mathematical development of DRAM and its calibration requirements are described in Appendix I to this report. The resulting equation for household location is as follows:

$$N^k_{it} = \sum_j E^k_{jt} \left[\frac{W^k_{it} f(c_{ijt})}{\sum_i W^k_{it} f(c_{ijt})} \right]$$

where

N^k_{it} = number of type k households located in zone i at time t

E^k_{jt} = number of type k employees working in zone j at time t

c_{ijt} = travel time between zone i and zone j at time t

W^k_{it} = multi-attribute measure of the attractiveness of zone i at time t to households of type k

[1] A. M. Voorhees and Associates (1972). "Application of the Urban Systems Model (U.S.M.) to a Region-North Central Texas", prepared for North Central Texas Council of Government, Dallas - Fort Worth, Texas.

The definition of the attractiveness measure is of crucial importance, and so will be discussed here. The zonal attractiveness measure consists of two principal parts. One part is the actual amount of land in the zone which is available for development, perhaps adjusted by its developability or existing level of development. The second part is the desirability, of the land in the zone as viewed by potential locators, apart from that solely due to its spatial location.

Many measures of the intrinsic attractiveness of residential zones have been proposed. These have included property value, quality of school systems, housing mix, degree of land use mix (e.g. other uses beside residential), and population mix. There was evidence in prior work by this author as well as by others that household incomes would be a good overall (or perhaps surrogate) measure of zonal attractiveness. Thus it was decided that the percentage composition of household types (in income quartiles) would serve as the attractiveness measure.

The amount of land available in the zone was measured in terms of vacant developable land. In order to adjust the attractiveness of vacant land to represent the presence or lack of infrastructure (e.g. sewers, water, electricity, etc.) vacant land is weighted by the percentage of developable land in the zone already developed. Finally, as a surrogate for the type of residential development, residential density was included.

Similarly, it was necessary to define a trip probability function. It is well known that most empirical trip distributions take the form of a normal curve considerably skewed to the left. It has been hypothesized that this distribution arises from the product of exponentially increasing numbers of opportunities (for trip satisfaction) encountered with increasing distance travelled, and exponentially decreasing propensity to travel each additional unit of distance. This is shown graphically in Figure 1. The trip probability function which results has the form

$$P_{ij} = (D^{\alpha}_{ij} \ \exp \ \beta D_{ij})$$

as described at the start of this chapter.

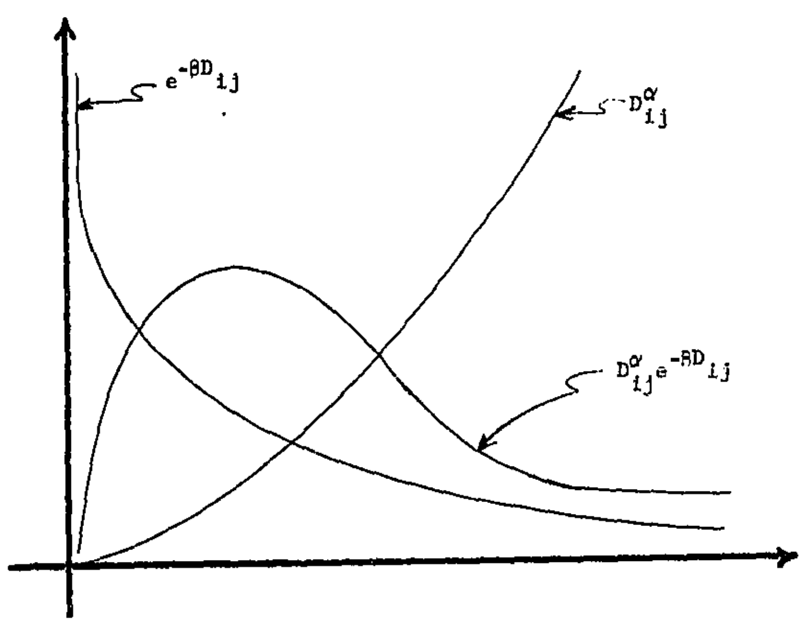


Figure 1: Trip Function Equation Form

The underlying hypothesis of the zonal attractiveness measure, developable vacant land weighted by its intrinsic attractiveness and its desirability, results in the use of a product function form. Thus the measure is written as,

$$W_i^k = (n_i^1)^{\delta 1} \; (n_i^2)^{\delta 2} \; (n_i^3)^{\delta 3} \; (n_i^4)^{\delta 4} \; R_i^{\delta 5} \; Q_i^{\delta 6} \; V_i^{\delta 7}$$

where

n_i^1 = percentage of zone i households which are in the lowest income quartile

n_i^2 = percentage of zone i households which are in the low middle income quartile

n_i^3 = percentage of zone i households which are in the upper middle income quartile

n_i^4 = percentage of zone i households which are in the upper income quartile

R_i = residential land area of zone i

Q_i = percentage of developable land in zone i which has been developed

V_i = vacant land in zone i

δn = a set of n parameters to be estimated

Thus, taking the trip probability function along with the zonal attractiveness measure, there are nine parameters to be estimated for each of the four household types.

Some model efforts have estimated parameters for a model's trip probability function, but have assigned values of 0.0 or 1.0 to any δ parameters (or their equivalents) in the attractiveness measure. Other model efforts have estimated values of the δ parameters by multiple linear regression, independent of the trip probability function. For an urban area with an existing spatial distribution of activities the parameters of these functions should not be separately estimated.

The reality which we are attempting to describe, like a solution of sugar and water, cannot be separated by mechanical means. The shape of the trip probability function is due, in part, to the spatial distribution of zonal attractiveness. Similarly, the attractiveness of a zone results in part from the households located therein. These have so located, in part, responding to the work trips which are an implicit aspect of living in that particular zone. Simultaneous estimation of the attractiveness parameters and the work trip parameters is thus required as a consequence of the inseparability of these phenomena.

A glance at the DRAM equations shown above makes it clear that standard parameter estimating procedures such as regression techniques are inadequate. At the other extreme, brute force trial and error methods may not yield useful results at reasonable cost. What is needed is a sophisticated n-dimensional search technique that doesn't make the assumptions necessary in regression, but is much more efficient than trial and error. Two candidate methods are pattern search and gradient search.

Holding, momentarily, the question of a proper criterion function, it may be assumed that one exists. In pattern search, successive explorations are made, as to the change in the criterion which results from a change in each of the parameters. Then, based on the information gleaned from these explorations a step is taken in all n-dimensions at once. In gradient search the gradient (an n-dimensional vector orthogonal to the mathematical surface, whose projection on that surface points in the direction of its steepest ascent) is evaluated at a given point and an n-dimensional step is taken in the direction indicated by the gradient projection. The gradient may be found using numerical approximations or by analytically solving the function's partial derivatives with respect to the criterion. It was not the purpose of this project to develop new calibration techniques. An efficient, operational, gradient search program was available and was used to estimate the parameters of DRAM. At some future date an investigation of alternative search procedures will be made.

Our inexperience with these techniques led to the aggregation of the San Francisco data set to 30 zones in order to lower the cost of our inevitable mistakes. A conservative approach was taken, with much attention paid to initial estimates (starting points for the search procedure) of the parameters. Experience with the technique has shown these concerns to have been unwarranted, the parameter estimates being easily done at reasonable computer expense. These preliminary efforts are further described in Appendix I.

Having once established the feasibility of the technique, the 108 zone Minneapolis-St. Paul data was approached. Again, no difficulties were encountered and the parameter estimates were readily obtained. The parameters and the associated criterion value, R^2, are shown in Table 7. No t- or F-values are provided, as these statistics, unfortunately do not apply to the non-linear equations of DRAM.

It is worth noting that the criterion function used in these estimations was the least-squares (sum of squared differences between observed and estimated data points) criterion. It was observed in the parameter estimations that the criterion surface tended to be rather flat in the neighborhood of the

Table 7: POPULATION COEFFICIENTS FOR DRAM - TWIN CITIES

	Distance		Household Composition				Land Conditions			
Variables	Opp'ty	Decay	L.I.	L.M.I.	U.M.I.	U.I.	Res.	% Dev.	Vacant	
Household Sector	α	β	$\delta 1$	$\delta 2$	$\delta 3$	$\delta 4$	$\delta 5$	$\delta 6$	$\delta 7$	R^2
Low Income 0-8000	1.04	-2.18	0.765	0.142	-0.558	-0.339	0.893	0.145	-0.031	0.927
Low Middle Income 8000-12000	2.11	-1.46	0.244	0.835	-0.373	-0.152	0.899	0.177	-0.044	0.900
Upper Middle Income 12000-16500	2.81	-1.31	0.086	0.157	0.500	-0.080	0.795	0.251	-0.079	0.900
Upper Income 16500 →	2.10	-1.44	0.131	0.099	-0.191	0.776	0.752	0.292	-0.033	0.905

best fit parameter estimates. Consequently the criterion was, in some cases, insensitive to small changes in some of the parameters. Furthermore, gradient search provides no information as to the statistical properties of the parameters found. A current investigation of the use of the maximum-likelihood criterion as an alternative to the least-squares criterion appears to be leading to resolution of both these problems.

Refering to Table 7, consider first the "Distance" or trip probability function parameters. The Low Income households show the lowest propensity to travel to work i.e. the largest negative β or "Decay" parameter. The Upper Middle Income households show the highest propensity to travel to get to work. Lower Middle Income and Upper Income households are much more willing to travel a given distance to work than is a Lower Income household, and not quite as willing as an Upper Middle Income household. These results are in accord with other empirical findings as well as with theories concerning the portion of a household's budget allocable to travel expense.

The opportunities encountered or "Opp'ty" parameter shows opposite results. As trip length increases Low Income households find the fewest opportunities for trip satisfaction. Upper Middle Income households find the most rapidly increasing numbers of opportunities with increasing trip length. In DRAM, trip satisfaction is the choosing of a residential location. Taking these results together, we find that Upper Middle Income households are likely to have the longest work trips, due not to a lack of residential opportunities but rather to a great willingness to travel to the "right place". At the same time, the Lower Income households will also have longer work trips but, in this case due to fewer residential opportunities, despite a greater unwillingness or inability to travel.

Turning next to the Household Composition variables, the first conclusion is that each income group is most likely to locate (or be located) in zones where it is already concentrated. The second conclusion is that any other household type is least likely to be located with concentrations of Upper Middle Income households. No other general conclusions may be drawn from these parameters.

Finally, turning to the Land Conditions, all household classes are positively affected by amount of residential land (Res.) and by percentage of developable land developed (% Dev.). There is a slight negative response to vacant land. The need for a statistical significance measure for these parameters is clearly needed here. Without it we cannot sort the meaning out of these last sets of parameters. As mentioned above, further work on this is now being done.

The values of R^2 achieved in the DRAM calibrations are higher than those of EMPIRIC. The numbers of zones are identical for both of these calibrations to the Minneapolis - St. Paul data. The dependent variables in the EMPIRIC calibrations are, however, "change-in-share" variables, while the dependent variables in DRAM are simply "share". One expects better data fits with share than with change-in-share variables. Thus it is difficult to compare these sets of results.

In order to better compare the models' performance, both the EMPIRIC and DRAM model packages were run form a 1960 base to a 1970 projection year. The 1970 model estimates were then compared to actual 1970 Census data. Due to data incompatibilities, only the household sections of the models were comparable. The results of this test, in terms of the correlations between the model estimates and the actual data are shown below (in terms of r^2).

Household Type	EMPIRIC	DRAM
LIQ - lower income	0.918	0.750
LMIQ - lower middle	0.941	0.828
UMIQ - upper middle	0.889	0.844
HIQ - upper income	0.829	0.699

From these, more comparable, evaluations it is clear that EMPIRIC achieves somewhat better fits to the data than does DRAM. Balanced against this is the fact of DRAM's more understandable and theoretically satisfactory equation structure, along with the empirical support derived from the signs and magnitudes of the fitted parameters.

Calibration of Employment Model for DRAM

The modelling of employment differs somewhat between the models being studied here. The EMPIRIC model incorporates six types of employment directly in its equation system and produces forecasts for each type as a matter of course. The IPLUM and DRAM models, as is the case with all Lowry derivative models, do not include a procedure for estimating "basic" employment, but require such estimates as an input. In the many applications of these models basic employment estimates have been generated in a number of ways ranging from educated guesses to rather complex models.* Consequently it was necessary to add a procedure for estimating basic employment to either DRAM or IPLUM in order to compare their performance to that of EMPIRIC. This section of the report describes the development of such a procedure.

As is described above in the section on calibration of the residential models, the initial calibration work for DRAM was undertaken with data from San Francisco, it was decided to begin work on an employment estimating procedure by using the same data sets. It was not the purpose of the project to develop new models, so the first thought was to use BEMOD, a model which had been developed with the San Francisco data.** The model used a large number of variables to describe each census tract in the region. The variables used were: slope, elevation, presence of navigable waterway,

* See Putman, S. H. "Intraurban Employment Forecasting Models: A Review and a Suggested New Model Construct", Journal of the American Institute of Planners, Vol. XXXVIII, No. 4, July 1972.

** Nathanson, J. "Basic Employment Model: A Model for Intra-County Location of Basic Employment and Land", BATSC Technical Report 222 (Preliminary), Bay Area Transportation Study Commission, Berkeley, California. (1970)

presence of rail facilities, a general accessibility measure, density of existing development, residential land, unused land, vacant land, and the distribution of employment types in the zone. The areal unit used was the census tract, but the parameters for the census tracts in each county or group of counties (six in the study area) were estimated separately. Ten employment sectors were used. The results of the regressions, done in May of 1968, for that model are tabulated in Table 8. Despite the many variables used, these results were not very good, particularly when compared to those obtained with the much simpler formulation used in the USM model for the Dallas - Fort Worth region.* The USM used little more than a lagged variable and an access measure to obtain much better data fits. Consequently an attempt was made to develop a similar set of simpler estimating equations for San Francisco.

The ten employment types of BEMOD were first disaggregated to twelve employment types. A number of regressions were estimated, using the two-hundred ninety-one zone areal system used in IPLUM. This areal system is an aggregation of the seven hundred seventy seven census tracts used in BEMOD, and further, was not broken into separate county regressions. The results yielded poor data fits. In an attempt to improve them, the seven manufacturing sectors were aggregated to three, resulting in a total of eight employment types. The three levels of sectoral aggregation used in these analyses are shown in Table 9. The results of these analyses, while in most cases as good or slightly better than the old BEMOD results were not satisfactory, the values of R^2 ranging from 0.35 to 0.58.

* Voorhees, A. M. and Associates "Application of the Urban System Model (USM) to a Region-North Central Texas", Prepared for North Central Texas Council of Government. (1972)

Table 8: BEMOD REGRESSION RESULTS (R^2), MAY 1968, TEN EMPLOYMENT TYPES, CENSUS TRACT

	MFG1	MFG2	MFG3	MFG4	MFG5	TRAN	WHOL	FIN	SERV	GOVT
San Francisco	.7098	.6376	.6344	.5543	.5636	.8725	.8725	.3921	.4710	.4969
San Mateo	.3107	.6469	.7763	.6772	.7444	.6413	.8993	.0571	.4521	.7693
Santa Clara	.2584	.1336	.0607	.0674	.0272	.0281	.1716	.1754	.3243	.2502
Alameda	.1078	.2091	.1712	.4958	.6459	.0927	.1672	.0546	.0565	.1381
Contra Costa	.0697	.1292	.0762	.0953	.2357	.0665	.2680	.1889	.5008	.2680
North Bay (Marin, Solano Napa, Sonoma)	.0938	.1681	.1622	.1128	.1082	.2361	.1457	.3522	.1058	.4412

MFG1 = Manufacturing, New Technology
MFG2 = Manufacturing, Centralized Urban
MFG3 = Manufacturing, Decentralized Urban
MFG4 = Manufacturing, Metal Fab. and Machinery
MFG5 = Manufacturing, Petrochems., Primary Metals

TRAN = Transportation
WHOL = Trade
FIN = Finance and Insurance
SERV = Services
GOVT = Government

Table 9

SECTORAL DEFINITIONS IN 4 EMPLOYMENT ANALYSES

	BEMOD* Sectors		EMPMOD 12 Sector Analysis		EMPMOD 8 Sector Analysis
1.	MFG 1	1.	Ag., For. Fish.	1.	Ag., For. Fish.
2.	MFG 2	2.	Mining	2.	Durable Heavy Mfg.
3.	MFG 3	3.	New Technology	3.	Durable light Mfg.
4.	MFG 4	4.	Centralized Urban	4.	Non-durable Mfg.
5.	MFG 5	5.	Decentralized	5.	Trade
6.	TRAN	6.	Metal & Machinery	6.	Fin & Ins.
7.	WHOL	7.	Petroleum & Prim. Met.	7.	Services
8.	FIN	8.	Transp.	8.	Gov't
9.	SERV	9.	Trade		
10.	GOVT	10.	Fin & Ins.		
		11.	Services		
		12.	Gov't		

* See Table 8 for definition of BEMOD sectors

Reference again to the USM work suggested that the most important single variable in their employment equations was lagged employment i.e. employment of the same type as that being estimated, in the same area, in the prior time period. This variable was not available for San Francisco which, incidentally, also precluded estimation of the parameters for the EMPIRIC model in that region. A double-logarithmic form of equation was tried instead of the traditional additive linear form, with a modest increase in the R^2 values. The two-hundred ninety-one zone data set was aggregated to ninety-eight zones in the hope of improving the ability to estimate employment in the area. These results were somewhat improved, showing values of R^2 ranging from 0.53 to 0.80 for the eight employment sectors. At this point it was decided that further work on employment estimates with this data base would be fruitless. This work confirmed our expectations as to what could be done in this vein with strictly cross-sectional data. The multiple regressions tested while not very good, produced results which were as good or better than the early results for the BEMOD regressions.

EMPMOD: The Minneapolis - St. Paul Estimates

The data set for Minneapolis - St. Paul contains employment data for two points in time. It is this fact which allows the calibration of the EMPIRIC model on this data and which, as will be described below, yields relatively good employment estimating equations.

The EMPIRIC model uses six types of employment, as follows:*

* Peat, Marwick, Mitchell & Co. "Calibration and Application of an "EMPIRIC" Activities Allocation Model for the Twin Cities Metropolitan Area", Final Report, Metropolitan Council, St. Paul, Minnesota, Dec. 1971.

1. MISC - miscellaneous (S.I.C. 01-17)
2. MFG - manufacturing and wholesale (S.I.C. 19-39,50)
3. TRANSP - transport., communic., utilities (S.I.C. 40-49)
4. RET - retail (S.I.C. 52-59)
5. SERV+FIRE - finance, ins., real est., services (S.I.C. 60-89)
6. LOGV+ED - local gov't., education (S.I.C. 82, 91-94)

Each of these types of employment was estimated in EMPIRIC with an additive linear equation. These estimated equations were shown in Table 5.

Given the EMPIRIC results, work was begun on estimation of the parameters of a set of equations for employment estimates to be used as input to DRAM. The equation form tested was additive linear, and a standard multiple regression estimation procedure was used. The variable definitions and equations are given in Tables 10 and 11. The same eight sectors defined for the San Francisco analysis were used for these Minneapolis-St. Paul parameter estimates. The areal system was the same one hundred eight zone system as was used by the EMPIRIC and DRAM analyses, and represented a slightly greater degree of areal aggregation than the two-hundred ninety-one zone system for San Francisco. The regression results, in terms of values of R^2, were as follows (for comparable sectors):

	Sector	EMPMOD	Sector	EMPIRIC
1.	Ag. Forest. & Fish	0.421	MISC	0.761
2.	Durable, Heavy Mfg.	0.758		
3.	Durable, Light Mfg.	0.811		
4.	Non-durable Mfg.	0.812	MFG	0.714
5.	Trade	0.866		
6.	Finance & Ins.	0.970	SERV + FIR	0.758
7.	Services	0.901		
8.	Gov't.	0.960	LOGOV	0.546

Table 10: DEPENDENT VARIABLES: EMPMOD

E(K)	=	Employment in industry group K (K=1, 8) in 1970
EMP(K)	=	Employment of type K, in 1960
TLA	=	(AVL-TL(I)**2: Variance of zonal total land
TLB	=	$\frac{\text{MAXTL - TL(I)}}{\text{MAXTL - MINTL}}$: Normalized " " "
BL	=	Industrial land (1960) + Available land (1960) in (I)
IDENS	=	Industrial density (1960) in zone (I)
POP	=	Res. population in (I) (1960)
RDENS	=	Res. density in (I) (1960)
DGRI	=	% change in # of households in income quartiles (1+2)
DGR2	=	% change in # of households in income quartiles (3+4)
DACIN	=	Change in composite accessibility to manufacturing emp.
DACCM	=	Change in composite accessibility to commercial emp.
DACG 1	=	Change in composite accessibility to income group 1
DACG 2	=	Change in composite accessibility to income group 2
DSWR	=	Change in Sewer system (land)
STE	=	% of total emp. in (I) in (1960)
SACTE	=	% of composite accessibility to TE. in (I) in 1960
SACHH	=	% of composite accessibility to HH in (I) in 1960

Table 11: ESTIMATED EMPLOYMENT EQUATION FROM EMPMOD

E1 = -0.0181 TLA + 0.0007 POP + 1331.4 DGR2 + 18629. DAGG1 - 15748. DACG2 + 8.5920

E2 = 1.1745 E1LAG + 770532. DACG1 - 1009028. DACG2 + 17931. DSWR + 28192. SACTE - 29.163

E3 = 1.3604 E3LAG + 15.848 RDENS + 23385. DGR1 + 23652. DGR2 + 109.96

E4 = 0.9491 E4LAG + 152642. DACIND - 107994. DACG2 + 166.76

E5 = 1.0515 E5LAG + 0.5846 IDENS + 0.0160 POP + 26253. DGR1 + 45314. DGR2 - 131.62

E6 = 0.8977 E6LAG - 0.1785TLA - 5.7198TLB + 1.9488 IDENS + 0.0035 POP
+ 4.6650 RDENS + 4471.2 DGR1 + 7411.3 DGR2 + 65497. DACG1 - 72701. DACG2
- 3969.4 DSWR + 4157.8 STE + 449.19

E7 = 0.6957 E7LAG - 0.4636 TLA - 14.782 TLB - 0.6809 IDENS - 0.0089 POP + 30452. DGR1
+ 26937. DGR2 + 290845 DACIND - 422560. DACCM + 85920. DACG1 + 16910. STE
+ 33841. SACHH + 1136.9

E8 = 1.5057 E8LAG - 0.7470 IDENS - 0.0062 POP - 28.378 RDENS + 13015. 13015. STE + 77.806

Note: All coefficients significant at 5% or better

There are several points to be noted here. First, as was the case in the residential models, these results are not strictly comparable as the EMPMOD dependent variables are static, while the EMPIRIC dependent variables are change in regional share. Second we note that the EMPIRIC and EMPMOD results are not strictly comparable due to different sector definitions.

There is a further point to be mentioned regarding numbers of zones. It has sometimes been asserted that for a given urban area and a particular model one should expect increasingly good fits of equations to data with decreasing areal disaggregation. In other words, ceteris paribus, fewer zones yields better fits to data. While it was not possible to specifically test this hypothesis in the study, the impression gained from working with both models on the various data sets is that this phenomenon, if it operates at all, is rather weak in its effects. In fact, while it probably operates at the high end e.g. a difference between 200 zones or 600 zones, it probably operates in reverse at the low end e.g. a difference between 100 zones and 10 zones. Further exploration of this phenomenon is planned for future work with DRAM.

Retail Employment Estimates in DRAM

Finally, it should be noted that in order to save projecc time the existing, uncalibrated, local serving employment model included in IPLUM was used during the sensitivity and policy tests of the DRAM package. This submodel should probably be replaced by a better "retail" model during future of the DRAM package.

Summary

Thus we have developed the two model approaches, EMPIRIC and the DRAM-

EMPMOD combination. EMPIRIC contains four household types and six employment types. DRAM-EMPMOD contains four household types and eight employment types.

All models were fit (i.e. parameters estimated) to the same 108 zone data base for Minneapolis-St. Paul, 1960 to 1970. EMPIRIC achieved a somewhat better fit to the data than DRAM-EMPMOD.

The focus of the work was on the residential equations of EMPIRIC, and on DRAM. The employment equations in EMPIRIC, and the whole of EMPMOD were simply necessary to provide inputs to the residential location estimates.

It is worth noting that a criticism often leveled at Lowry derivative models, e.g. DRAM, is that they depend on basic employment estimates as inputs. These inputs, it is contended, can never be perfect and therefore must have an adverse effect on the residential estimates. A test run of DRAM was made with EMPMOD inputs and an alternative run was made with actual employment data inputs. The subsequent two sets of residential outputs were compared to actual residential data. The differences in the correlations between the estimates and actual population were not statistically significant. Further, it should be recognized that the outputs of the employment equations of EMPIRIC, however perfect or imperfect, are inputs to the residential equations just as the outputs of EMPMOD are input to DRAM.

Finally, though EMPIRIC achieves better fits to the data, it lacks theoretical form and its estimated parameters do not agree well with theory, intuition, and other empirical findings concerning urban spatial phenomena. DRAM's parameters do agree with these, but do not result in as good a fit to base data as was found with EMPIRIC.

CHAPTER THREE: TESTS OF MODELS

Introduction

The previous chapter's comparison of the two models ends inconclusively. The EMPIRIC model achieves somewhat better fits to the data, but leaves much to be desired in the way of theoretical underpinnings. The DRAM package has stronger theoretical underpinnings, which are supported by the empirical results, but does not achieve as good fits to the data as does EMPIRIC.

In this chapter the models are tested and evaluated in a different way. By models we mean, EMPIRIC on the one hand, including all population and employment sectors. The DRAM package, on the other hand, consists of DRAM and EMPMOD as described in the previous chapter. Each of the models, again using a common data set, is subjected to a wide variety of changes in inputs. The resulting changes in output are then compared both to each other and to expectations based on theory and existing evidence as to the behavior of urban spatial systems.

Each type of test was done by making identical input changes to both models and observing the resulting changes in output. In some cases further tests were made of one model and not the other in order to further investigate a particular question.

The tests described below fall into several types:

1. Changes in population
2. Changes in accessibility

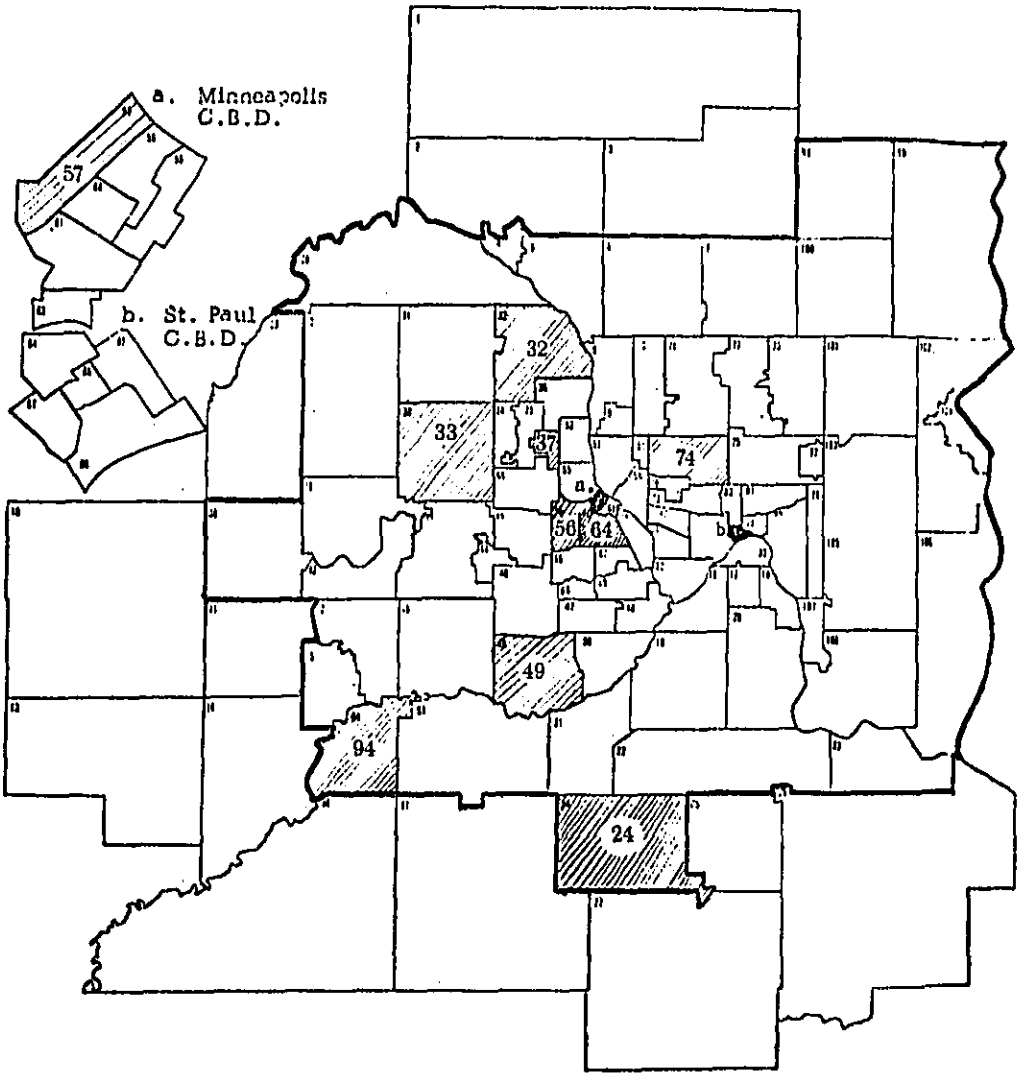
3. Changes in employment

4. Changes in land use

Each of the tests involved changing input data to the models, and observing corresponding changes in outputs. All tests were made using the Minneapolis-St. Paul data base, with 1960 as the base year and 1970 as the future year. All results (i.e. test run outputs) were compared to a control run of each model (hereafter referred to as CR). The CR was also, 1960 to 1970, but no inputs were manipulated. Some tests can be construed as representing possible policy alternatives, while others are clearly just manipulations of the models. The tests described, a subset of all those done, rather clearly illustrate the model's capabilities. Many additional tests might have been run but, as the reader will find by the end of this chapter, they were unnecessary for the purposes of this project.

Changes in Population: Low Income Household Reductions

These tests were made for several different zones. One example was for Zone 57 in the Minneapolis urban core (see Map 1). In this zone a simplistic form of "Urban Renewal" was done, all low income households were removed in the base year. This amounted to 697 low income households, or 79.6% of the households in Zone 57 in the base year. The EMPIRIC base year input, being expressed in shares (as discussed in the previous chapter) was changed so that the



MAP 1: MAP OF TWIN CITIES REGION WITH ANALYSIS ZONES SHOWN

(Minneapolis and St. Paul CBD areas shown enlarged) region's share of low income households in Zone 57 was zero.

The responses of the two model packages to this input change were quite different. EMPIRIC showed virtually no response. Differences between this test run and CR were in no case more than 0.05% of CR. This result is contrary to one's intuitive expectations. Reference to the EMPIRIC equations in Table 5 of Chapter 2 will provide an explanation. Base year low income households (LIQ) appear only in the first equation, and would have the value 0.0044 (LIQ for Zone 57 divided by the region's total LIQ) in the CR. Given this value, the coefficient of -0.415, and the additive form of the equation, deletion of LIQ will produce only a very small change in ΔLIQ. This very small change appears on the right side of the equations for the other population types and government employment. In each case, the fractional coefficient and the additive form of the equation suggest that differences from CR will be minimal. In fact, they turn out to be negligible.

The DRAM response to this input change was quite a different matter. Compared to CR, the low income households in Zone 57 were down 65% in the projection year. Lower middle income households were down 42% in Zone 57. At the same time upper middle income households increased 16% while upper income households showed a 15% increase. The absolute sizes of the two upper income household class increases were, however, rather small, so the zone showed a net household decrease of 47% compared to CR. These results are very like actual metropolitan experiences with renewal attempts. Renewed center city zones often remain relatively empty for a time, with perhaps some increases (significant percentages but small absolute numbers) in upper and upper middle income households.

It is worth noting where the displaced households relocated. Of the 697 low income households removed from Zone 57 in the base year, 241 returned to that zone. Of the remainder 194 relocated in a ring of zones adjacent to Zone 57, mostly in those zones with prior large numbers of low income households. The remainder of the low income households relocated throughout the region, with emphasis on the next adjacent ring of zones and in two zones adjacent to the St. Paul urban core; in all cases, zones with relatively large numbers of low income households in the base year. (Table A1)[1]

A second DRAM run was made, differing in that the deleted low income households were not allowed to return to Zone 57. The pattern of these results was identical to those of the previous test. In all cases the results simply showed slightly larger responses. (Table A2)

Pursuing this line of experiment with DRAM, a second pair of test runs was made for Zone 49. This zone, in the CR was a rather rapidly growing upper-middle and upper income suburb. In the base year 384 low income households, 7.5% of the total households in the zone, were deleted for the test run. By the projection year there were 928 low income households, making 8.3% of the new total. In CR the corresponding figures are 1046 and 9.2%. For the projections of other household types in the test run the respective percentages were lower-middle income 19.0%, upper-middle income 34.6% and upper income was 38.2%. In CR the corresponding figures were 20.8%, 34.3%, and 35.7%. Thus, the elimination of a small low-income enclave in the base year did not prevent growth of low income households in in the zone, but did slightly retard that growth. (Table A3)

The test was rerun, with the same deletion, but precluding the return of low income households (perhaps simulating large lot zoning?). In this case there was a net decrease in the zone's population of 9.9% compared to CR. There were no low income households, a 23.0% decline in lower-middle income

[1] Table references at the ends of test run discussions are to tables included in Appendix II, which tabulate selected, relevant, run outputs.

households, a 0.2% increase in upper-middle income households, and an 11.3% increase in upper income households. The low income households which were prevented from locating in Zone 49 all located in the zones falling between suburban Zone 49 and the Minneapolis urban core. Apparently the exclusion of low income households from this suburban zone has the effect of preventing those households from leaving the urbanized area immediately adjacent to the city's core. (Table A4)

Having tested "urban renewal" in an urban core zone, and in a suburban zone, one more DRAM test was made, for Zone 56, midway between the core and the suburbs. In this case, 1000 low income households were removed from the base year. This was a 10.2% decrease in the low income households and 4.8% decrease in the total households in Zone 56. In both the test run and CR the zone's households increased substantially from the base year to the projection year. The test run showed fewer low income and low-middle income and more upper-middle and upper income households than did CR. These four household types showed growths of 82.9%, 56.7%, 26.1%, and 1.8% in the test run and of 102.6%, 67.6%, 25.7%, and -2.9% in CR. Thus again, a decrease in low income households in a zone in the base year retards the growth of the lower two income groups, and slightly accelearates the growth of the two upper income groups. Finally, even though the deleted low income households were not prevented from returning to the zone, most relocated in adjacent zones slightly closer to the urban core. (Table A5)

As a last test run in this series, a run was made where 1000 low income households were deleted and 1000 upper income households were added to Zone 56 in the base year. Comparison of this test run to CR yielded differences almost exactly double those found between the previous test run and CR. (Table A6)

To summarize the results of the first in this series of tests: 1) EMPIRIC shows no response in either population or employment, 2) DRAM shows excellent population response and almost no employment response. EMPIRIC's lack of response is not surprising given its equations and parameters, but is inconsistent with current theoretical and empirical findings. DRAM's response was sufficiently interesting to suggest a further set of tests. On their conclusion it appears that the population responses of DRAM are quite in keeping with our current understanding of the actual phenomena being simulated. DRAM's lack of employment response is perhaps explained by the fact that the population changes, while significant, are of small absolute magnitude and therefore do not stimulate a noticeable employment response.

Changes in Population: Low Income Household Increases

In the same way that a decrease in a zone's low income households may be used to crudely describe a simplistic form of urban renewal, an increase in a zone's low income households may be used to crudely describe a public housing project. A series of test runs was undertaken to study this phenomenon.

A zone midway between the urban core and the suburbs (Zone 37, see Map) was selected for the first of these tests. One thousand low income households were added to this zone in the base year. Once again, EMPIRIC showed virtually no response to this change in inputs. DRAM, once again, responded in a way consistent with theoretical and empirical findings by others.

In Zone 37 DRAM showed projection year increases, compared to CR, of 41.1% in low income households, 30.5% in low-middle income households, and

1.1% in upper-middle income households. Upper income households showed a 5.4% decrease compared to CR. Overall, Zone 37 showed an 11.3% increase in total househols in the test run, compared to CR. Thus the addition of the 1000 low income households in the base year (a 21.2% increase in the zone's base year total households) yields a rather strong tendency for the zone's household composition to change. In the control run, the composition is 18.0%, 22.3%, 29.2% and, 30.4% low income to high income respectively, and in the test run the composition is, 22.8%, 26.2%, 26.5%, and 24.4% for the four income groups, low income to high income respectively. Finally, we note that the household types which increased in Zone 37 were drawn from a ring of adjacent zones, and the upper income households which left Zone 37 dispersed to the ring of adjacent zones. (Table A7)

To further explore this phenomenon two further DRAM test runs were made for two suburban zones. In the first of these runs Zone 32 received an increment of 1000 low income households in the base year, and in the second run the increment was put in Zone 33 instead. The results in both these runs were virtually identical. Comparison of the test runs to CR showed increases in low income and low-middle income households and decreases in upper-middle and upper income households. For Zone 32 the percentage composition of households was 10.0%, 24.3%, 34.8%, and 30.9% (low to high income) in CR and changed to 13.9%, 31.4%, 31.0%, and 23.9% in the test. For Zone 33 the figures were 9.1%, 24.2%, 35.7%, and 31.0% for CR and 12.7%, 31.5%, 31.9%, and 23.9% for the test. Thus the introduction of low income household increments to suburban zones in the base year produced long term changes in the zones household composition. (Table A8)

A final test run of a low income household increment was made for a rural zone. Zone 94 is a rural zone which showed a net decline in population from the base year to the projection year. The bulk of this decline was in the low income households who dropped 66.4% from the base year to the projection year. During the same period the high income households increased 43.5% in the zone. The addition of 1000 low income households to this zone in the base period only partly altered its situation in the projection year. Low income households still declined, though by a somewhat smaller 47.4%. Lower-middle income households grew by 25.8% compared to a decline of 11.6% in CR. Upper-middle income was relatively unchanged, growing by 28.7% in the test run and by 24.5% in CR. Finally, high income household grew by 28.0% and 43.5% in the test run and CR respectively. (Table A9)

To summarize the results of these tests, where low income households are added to a zone, we find EMPIRIC not responding and DRAM responding as expected. Adding low income households in the base year changes the zone's projection year household distribution. The shift is in the direction of increases in low and low-middle income households and decreases in high and high-middle income households. The extent of the shift depends on the initial total population and population composition of the zone.

Changes in Population: Upper Income Increases

As a final set of DRAM population tests, two runs were made where upper and/or upper-middle income households were added in the base year. In the first of these tests 1000 upper income households were added to Zone 74 in the base year. This is a populous zone, well within the urban area, but not in the core. These new households represented a net increase of 16.7%

and a 40.0% increase in high income households. The result of this change was that the high income households in Zone 74 grew somewhat more in the test run than in CR, and all other income classes grew somewhat less. (Table A10)

The second of these runs was a test of Zone 94 (a rural zone used above for a low income increment test) in which an increment of 1000 high income plus 1000 high-middle income households was added to the zone in the base year. This produced results similar to those produced by the high income household increment, but not quite so pronounced. (Table A10)

Changes in Population: Summary

EMPIRIC shows no response to exogenous changes in base year population (households) DRAM shows responses consistent with both theoretical and empirical descriptions of urban phenomena. This is amply demonstrated by an extensive series of DRAM tests. Briefly stated, increases in low income households in a zone produce decreases in high income households, decreases in low income households produce increases in high income households, and increases in high income households produce decreases in low income households. Ripple effects are often observed in the ring of zones adjacent to the zone in which the test was effected.

Changes in Accessibility: Regionwide

Measures of the ease or difficulty of interaction between activities are central components of virtually all urban spatial models. This interaction phenomenon is contained in the several accessibility variables found in EMPIRIC, and is an integral part of the DRAM formulation in the form of the trip probability function. The next series of model runs described were intended to evaluate the models' response to changes in this variable.

Throughout the literature on transportation and urban development one finds the observation that where transportation is readily available and consequently, access is great, development tends to be spread out. Similarly when access is poor, development tends to be more concentrated and, in the case of larger regions, subnucleated. Recent experience throughout the United States amply demonstrates the generality of this phenomenon, with virtually every transportation improvement being closely followed by further spread of activities.

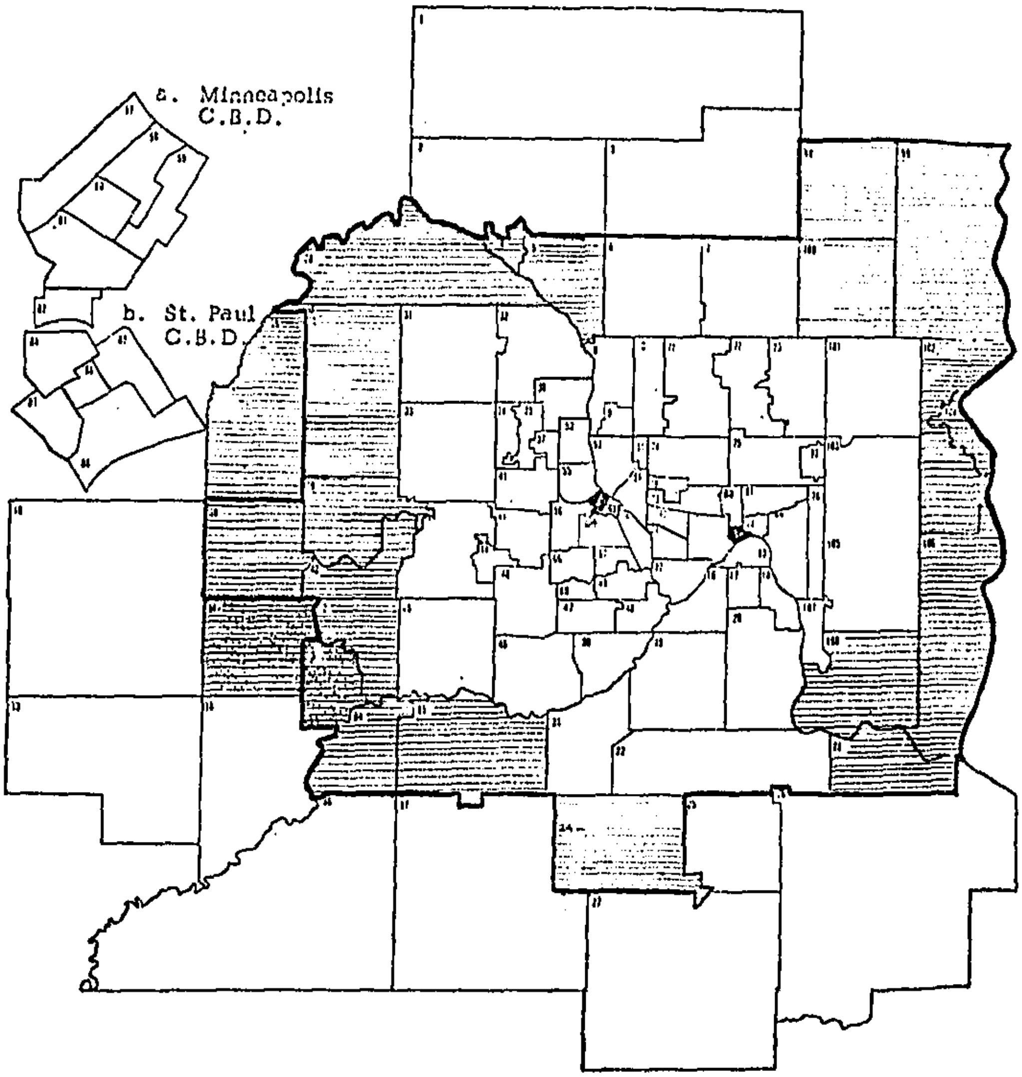
The first tests of the models' performance in response to access changes were with respect to regionwide changes. These tests might, for example, represent significant increases or decreases in fuel cost and/or availability. In the first run there was an arbitrarily imposed increase in impedance (i.e. an increase in travel time and/or cost and a subsequent reduction in accessibility). EMPIRIC showed no response to this input. Before discussing the DRAM response it may be useful to describe the mechanics of implementing these changes in the models.

In both models the initial datum is a zone-to-zone matrix of travel times or costs (or some composite figure). For the Minneapolis-St. Paul data base the interzonal travel times, estimated for an unloaded (uncongested) network were used in the models. While the congested travel times are preferable for these models, they were not available for use in this project. For use in DRAM, these matrices are simply one of the data inputs and are used directly in the running of the model. Changes in these impedances may be implemented by actually modifying the dataset or by adding the modification to DRAM's input routines, thus modifying the data as it is read in. In either case, a regionwide increase or decrease can be accomplished by multiplying by 1.1 or 0.9. Parking charges can be imposed by adding to the vector of impedances terminating in the zones where charges are to be instituted. Improvements between various zones and other zones can be imposed by multiplying the appropriate and/or columns of the impedance matrix. All in all, changes in impedance for DRAM are easy and direct.

In EMPIRIC, the procedures are more complex. The model package contains many programs for manipulations of data inputs and outputs. Modification of the inputs to EMPIRIC involves several steps of processing data through various programs. For impedances, there are many processing steps resulting in the several accessibility measures. Each of these measures is a vector of length n (n equals the number of zones). Finally, these vectors are converted to regional shares, i.e. scaled so their sum equals 1.0 exactly. It is precisely this scaling that results in EMPIRIC's nil response to regionwide changes in impedance, zone specific changes do produce responses as will be discussed below.

DRAM showed a consistent response to the first run of this set, which involved a regionwide 10% increase in impedances i.e. highway times and/or costs. The low income household response to this was mixed, with some fringe areas showing declines compared to the CR, and some showing increases. The other three income classes were, however, uniform in their relative decreases in the urban fringe zones. The implication here, are which certainly needs further study, is that ceteris paribus, low income households are the least sensitive to travel costs. It may be not so much a matter of insensitivity to changes in travel cost as a matter of inability to respond to those changes, due to other factors such as housing discrimination or limited employment opportunities. This, of course, ties in with the interpretation of the distance parameters discussed in the previous chapter. The net effect on all households is that 15% of the regions zones, all located at the urban-suburban fringe, showed relative declines of 10% or more compared to CR. At the same time, while less marked, employment showed some degree of centralization and a good deal of churning in the core and near core. Map 2 shows the zones with 10% or more decline in total households, the declines were absorbed (i.e. matched by increases) in the urban cores.

The second run of this set involved a regionwide 10% decrease in impedances. Again, DRAM's low income household response was mixed, with equal numbers of fringe zones showing gains or declines compared to CR. The lower-middle income households showed a strong tendency towards decentralization, with gains of 10% or better in a ring of fringe zones completely surrounding the metropolitan area. Upper-middle income households and high income households also showed strong decentralization response to this regionwide impedance decrease. At the same time, Basic 1 employment declined in the urban core, Basic 2 employment showed signs of beginning suburbanization, and Non-basic employment declined in the urban core. The net effect of these impedance decreases is a substantial decentralization of population and the beginnings of decentralization of



Map 2: ZONES SHOWING GREATER THAN 10% DECLINE IN ANY HOUSEHOLD SECTOR DUE TO REGIONWIDE IMPEDANCE INCREASE

employment, as compared to CR. Map 3 shows zones with a 10% or more increase in total households, the corresponding declines were all in and adjacent to the urban core. In short, relative decreases in transportation costs encourage further urban sprawl, while relative increases in transportation costs discourage sprawl and perhaps even encourage re-centralization.

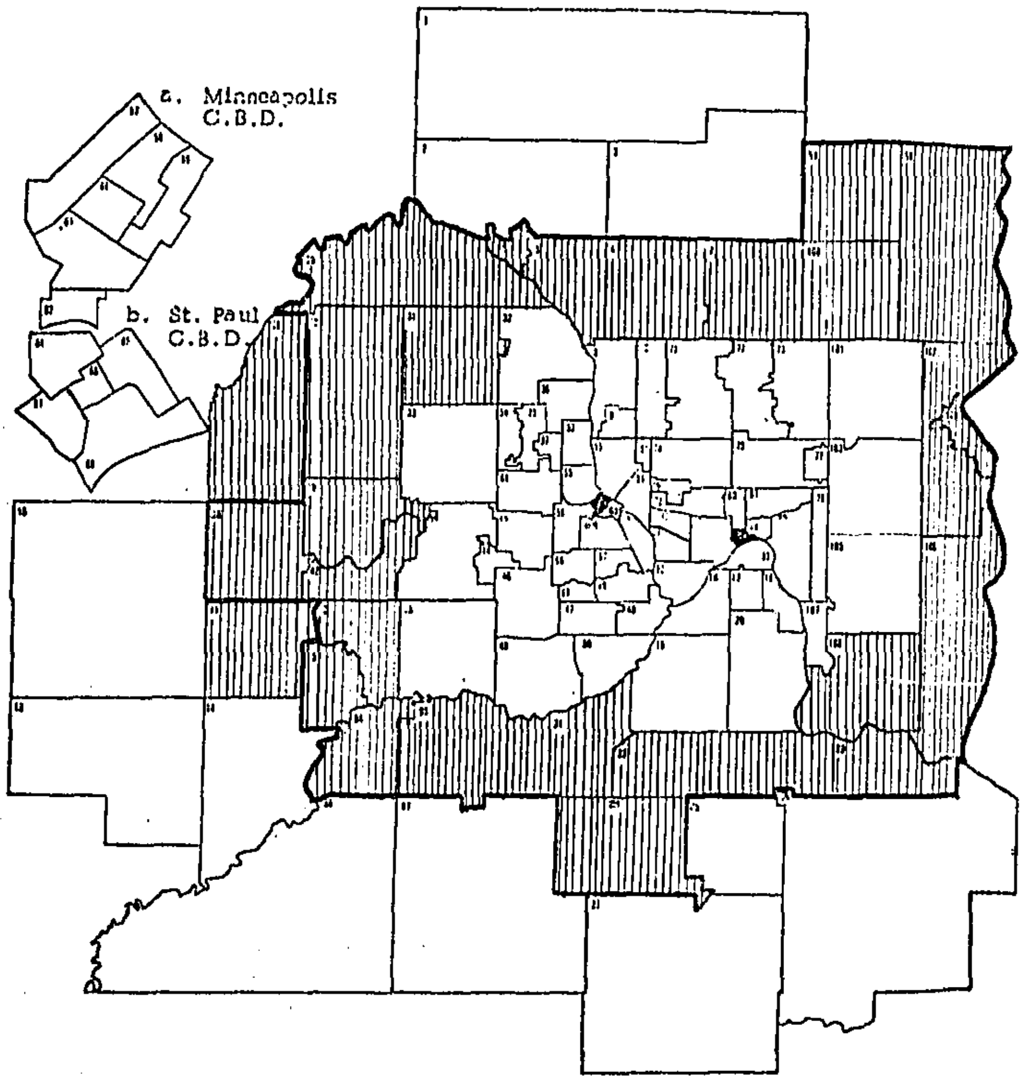
Changes in Accessibility: Zone Specific

The second group of access tests involved changes in impedance between specific zones and the rest of the region. The first in this group of tests involved a small improvement in the accessibility, to the rest of the region, of a zone on the suburban-rural fringe (Zone 24). This improvement was in the form of a 5% reduction in impedance from that used in the CR. Theoretically one would expect modest increases in population and employment in the affected zone. Since their impedances (access) to the region would remain unchanged, there would be little or no spillover effect to surrounding zones. We note that an actual transport improvement to one zone would simultaneously affect others because the network would connect many zones to each other via the improved link(s). In an integrated transportation and land use model this could easily be simulated as the transportation system is described and used in link-by-link form.[1] This was not possible in the present project due to the use of impedance data in lieu of the actual networks.

The EMPIRIC response to this test was minimal. No population or employment sector changed as much as 1% from the CR.

The DRAM run showed responses more in accord with expectations. All household types showed increases between 10% and 19%, with total households

[1] Putman, S. H. (1974) "Preliminary Results from an Integrated Transportation and Land Use Models Package", Transportation, Vol. 3, pp. 193-224.



Map 3: ZONES SHOWING GREATER THAN 10% INCREASE IN ANY HOUSEHOLD SECTOR DUE TO REGIONWIDE IMPEDANCE DECREASE

in Zone 24 increasing by 13.3% compared to CR. Total employment in the zone showed a slight increase of 1.1% compared to CR. The non-basic employment showed the greatest increase, 9.4% compared to CR. There were virtually no effects in surrounding zones, as the absolute change in households in the zone was only 221, with only 14 more employees. (Table A11)

The second test of this pair involved a 20% improvement in accessibility of the same zone (Zone 24) to the rest of the region. The expectation was that the results should be simply an amplified version of those from the previous test.

The EMPIRIC results were still minimal. The population changes were still less than 1%, though the total employment for the zone did show a 3.7% increase. The DRAM results showed a 67.9% increase in total households and an 8.0% increase in total employment. Though these changes seem large, it must be remembered that a 20% improvement in accessibility of an individual zone to the entire region is a phenomenal increase in accessibility; almost equivalent to replacing an unpaved road with an expressway. (Table A12)

The same pair of tests was then repeated for an urban core zone (Zone 64). The first of these runs was for a 5% accessibility improvement for the zone. The EMPIRIC response to this change consisted of very slight declines in the lower two income classes and modest increases in the upper two income classes. Total employment increased by approximately 2% in the zone. This is the first EMPIRIC run to show any noticeable response. The results are in accord with what one expects from looking at the equation coefficients, but not quite what might be expected from other theoretical and empirical findings. It is not clear why improved accessibility of an urban core zone should cause a decline in its largest population groups. Typically when expressways have connected to urban core areas the areas have experienced declines in upper income groups and increases in lower income groups.

The DRAM response to this change in accessibility was a 4.4% decrease in upper-middle income households, increases of approximately 2% in low-middle income and high income households, and a substantial increase of 20% in low income households. These results are more in accord with our expectations. Employment in the zone shows a total increase of 10.3% compared to CR. (Table A13)

Rerunning these tests with a 20% accessibility improvement produces similar, but stronger responses in all cases. We note that in the EMPIRIC run the decline in low income households was taken up by adjacent urban zones and similar zones in the St. Paul urban core. The increase in high income households was at the expense of suburban zones running northwest from the Minneapolis urban core. In the DRAM runs the low income increment was drawn from adjacent urbanized zones, while the upper-middle income decline was taken up by several suburban zones. (Table A14)

In summary, DRAM is again more responsive than EMPIRIC, in this case to changes in accessibility. EMPIRIC shows no response whatever to regionwide changes in accessibility. DRAM shows increased urban sprawl or decentralization with regional improvements in access, and decreased sprawl or centralization with regional access decreases. When access to a specific zone is increased, DRAM shows increases in population and employment. If the improvement was for a suburban zone, all types of population increased. When the improvement was for an urban area the principal increase was in low income households, with a decline in upper-middle income households. All in all, even though this was the first set of tests to evince any response from EMPIRIC, the response did not seem to be correct. DRAM, again gave the proper response though we have some minor reservation as to whether it may have been an over response.

Changes in Employment: Basic

A third set of tests of the models was run in the form of changes in base year employment in a zone. In each case an arbitrary increase was added to the base year employment in a particular zone. The regional forecasts for the projection year were not changed, so that any projection year increase, compared to CR, in one zone was at the expense of some other zone. A large number of test runs of this sort were made, but only some of them, enough to describe the model's responses, are discussed here.

Again, an urban and a rural zone are described, the same zones (Zones 64 and 24 respectively) as were described in the accessibility tests.

Taking first a 10% increase in basic (the sum of BASIC 1 plus BASIC 2) employment for Zone 24, EMPIRIC shows virtually no response. DRAM shows virtually no population response, and a total employment increase of 3.3% for the zone. Thus in both cases the 10% basic employment increment was dispersed throughout the region with no significant effect. (Table A15)

The second set of runs had a 30% increase in basic employment in the base year for Zone 24. EMPIRIC again shows virtually no response. DRAM shows a total population response of less than 1% increase above CR. There is a net employment increase of 10%. This looks small until realizing that the 30% increase in the base year was 84 BASIC 1 employees and 5 BASIC 2 employees. This yields an increase of 113 BASIC 1 and 8 BASIC 2 employees over CR for the projection year. This is a multiplier of 1.35 and 1.60

for each of these employment types. These changes were so small as to have negligible effects on adjacent zones. (Table A16)

A similar pair of test runs was made for the urbanized Zone 64. The DRAM run showed increases of 13.6% and 12.4% for BASIC 1 and BASIC 2 respectively in response to the 10% basic employment increment. But as basic employment is only 25% of the zone's employment, the total employment in the zone shows a net increase of 2.3% compared to CR. Population therefore shows virtually no change. (Table A17)

The EMPIRIC run of the 10% test showed no population change and an employment change of 14.4% increment to BASIC 2. The EMPIRIC run of the 30% test showed no population change and a 43% increase in BASIC 2. (Table A17)

The DRAM response to the 30% test showed small changes, less than 1%, in population and 41% and 37% increases in BASIC 1 and BASIC 2 respectively. The net employment increase for the zone was almost 7%. There were modest increases in low income households in all the zones adjacent to the Zone 64 test zone, at the expense of urban core zones in both Minneapolis and St. Paul. (Table A18)

Changes in Employment: Non-Basic

A similar set of runs were made with changes in non-basic employment in specific zones in the base year 1. These results were similar to the Basic employment runs. DRAM showed small (almost negligible) changes in population, and employment changes mostly made up of the exogenous change. EMPIRIC showed no population response and conflicting patterns of employment changes. (Tables A19-A22)

To summarize the results of changing base year employment, neither model shows much response to significant percentage increases in employment if they are not significant absolute increases as well. DRAM does demonstrate an employment multiplier effect, in that a base year increase of X% basic employment yields a projection year increase of $(1+\alpha)X\%$. EMPIRIC shows no response when suburban or rural zones are tested. In urban areas only BASIC 2 employment changes in the projection year even though both BASIC 1 and BASIC 2 were changed in the base year. EMPIRIC in no test shows any population response. In DRAM significant population response resulted only from large employment changes and was principally a matter of low income household increases in zones where there had been large employment increases.

Changes in Land Availability

The last group of runs to be discussed here involved tests of several degrees of land conservation policy. Each of these runs adopted a different policy as to the definition of open space for preservation. In effect, each of these runs removed different amounts of land from the available developable land in each zone.

The first set of runs deleted floodplain areas from available developable land. These areas were, of course located along the various rivers and streams that flow through the area.

The EMPIRIC run of this test showed moderate increases of low income households in older, but not core, urban areas. There was no significant movement of lower middle income households, while there were large declines in the upper two income groups for those same urban, but not core, areas. The low income increases were drawn from throughout the region, and the two upper income decreases were up in zones throughout the region. The employment response was mostly a matter of modest churning movements throughout the region.

The DRAM run of this test showed modest decreases in all income classes, throughout the region, taken up by large increases in zones adjacent to the urban core areas. The lowest and highest income households were least affected, with only modest changes. The two middle income household groups showed more substantial changes. Employment showed modest churning throughout the region.

The second set of runs deleted both floodplains and aquifer recharge areas. The sum of these began to be a substantial amount of land. Both the EMPIRIC run of this test and the DRAM run of this test showed considerable churning of both households and employment. A detailed, zone-by-zone, analysis of these runs is the only way to properly describe the results since the policy protected acres do not conform to the more traditional urban vs. suburban or rural sorts of descriptions. Suffice it to say that constraints on the use of land of the magnitudes involved here, produce rather substantial locational effects throughout the region.

In the next set of runs the land policy was even more restrictive, prohibiting use of floodplain, aquifer recharge areas, and wetlands. As would be expected both EMPIRIC and DRAM responded to this policy with even more churning of households and employment. It is interesting to note that their responses were very different, one from the other, even to the extent of being almost opposite. Closer investigation revealed that while the response was smaller than would be expected, the EMPIRIC responses were in the proper direction. The DRAM responses were, in some zones, backwards.

This was traced to discrepancies in the policy descriptions which attempted to preserve more land than was available. Consequently it was not possible to fully evaluate the DRAM responses to these policies in this study. Subsequent work with the model has corrected these problems so that these policy tests will eventually be properly examined.

CHAPTER FOUR: CONCLUSION

When, in the early 1960's, the first urban computer simulation models were being developed, one of the principal goals was to develop the capability of assessing the consequences of various urban renewal plans on the spatial distribution of activities. It was hoped that different public policies capable of altering the mix of activities in a zone could enter the models in various forms. The arrival or departure of an employment facility would induce significant effects in the model outputs. The arrival of a number of households of a particular income class might well result in changes in location of other households and perhaps of some employees too. Similarly the departure of a group of households would probably further, induce changes in a zone's activity mix.

Further, it was hoped that the density and degree, or extent, of development in a zone would also be affected by policy inputs. Clearance of certain types of structure would change density as would the erection of new structures. The construction of large new development, say of single family residential homes, or at a different density -- of apartments, would change both the zone's density as well as its extent of development. These changes would induce other changes, both in employment and in population location. In a related way, changes in the amount of land available in a zone should affect future location of activities in a zone. More stringent land use controls, having the effect of reducing available land, will change the pattern of activities locating in a zone. Similarly holding back land from development should also result in changed location patterns.

Finally, the spatial separation of activities from each other was expected to be a key variable in these models. This variable is usually expressed in terms of travel times and/or travel costs between zones and activities. Thus any substantial change in the transportation facilities should result in a change in activity distributions.

Many modelling projects were begun, with very few being successfully completed. It was a chaotic time for urban modelling. Each model had its proponents who claimed that their's was "the way". Not many of these models have survived, though there are occasional uses of one-time-only models or newly developed ones. The majority of recent model applications have been of either EMPIRIC or Lowry derivative models, with basic research efforts being performed independent of ongoing applications. Thus it seemed to be a good time to assess these two most-used models and to subsequently provide some guidance as to future applications of existing models as well as to directions for future research efforts.

The results of this project are quite clear. EMPIRIC achieves good fits to base data, but is not adequately sensitive to changes in input variables. This is probably due to its lack of an explicit theoretical form. The model has, however, been very useful for shorter term urban projections and it should be remembered that at first, even its authors claimed associative validity, rather than any genuine theoretical validity.[1]

The best of the Lowry derivative models in current U.S. use would not have compared especially well to EMPIRIC. Its theoretical structure is rudimentary, its disaggregation of population types is accomplished

[1] Hill, D.M. (1965) op.cit.

independent of the location procedure, and it relies on several exogenously defined constraint mechanisms to achieve its relatively good fits to base data. Finally, there was no standardized procedure for a statistically valid estimation of its parameters. Taking a cue from current British modelling practice, the model was reformulated in a more theoretically correct form so as to 1) allow for explicit disaggregation of the population as part of the location process, 2) eliminate the need for the exogenous constraint procedures, and 3) define the proper method for estimating its parameters. At the same time, the model differed from current British practice by making use of a multivariate zonal attractiveness measure, a particular feature of many of the U.S. developed Lowry derivatives.

This new model, called DRAM, did not fit the base data quite as well as EMPIRIC. However, its response to changes in input variables was excellent. This, coupled with the empirical confirmation of the model's form by its parameter estimates, indicates that it would give more accurate forecasts than EMPIRIC. This is expecially the case when the forecasts are of responses to policy inputs.

Current research with DRAM is proceeding in several directions. First, attempts are underway to routinize its parameter estimation procedure. This procedure, which utilizes mathematical search procedures, is no more complex than multiple regression, but is less well known and thus may cause some apprehension in potential users of the model. It is hoped that several case study applications of the model will help to ameliorate these problems. Second, further testing and improvement of the model itself is underway, including its fitting to as many different data sets as possible in order to test its consistency for different urban areas. Finally, DRAM has been incorporated in the Integrated Transportation and Land Use Package - ITLUP as a part of an ongoing research effort.[1]

It is perhaps only a little presumptuous to suggest that this work be used to mark the end of an era. For policy sensitive forecasting applications it would seem to be difficult to justify using anything other than a Lowry derivative model, perhaps DRAM, and calibrating it by the procedures discussed in this report. For future research efforts, it seems reasonable to suggest attempts to extend the theoretical structure of this model in the direction of bridging the gap to micro-economic theories of behavior on the one hand and in the direction of fuller integration of models with planning processes on the other. Were these suggestions to be followed, applied models would reflect the most advanced techniques practical for planning purposes at the same time that their results would provide feedback in the form of empirical results which could influence ongoing research in urban spatial dynamics.

[1] Putman, S. H. (1974) op.cit.

Appendix I

DRAM paper

Calibrating a Disaggregated Residential Allocation Model - DRAM

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to be presented at:
Regional Science Association - British Section
Eighth Annual Conference
London, Sept. 4-5, 1975
(Revised Dec. 1975)

ACKNOWLEDGEMENTS

This work was supported by the National Science Foundation, and in part by the U.S. Department of Transportation, via Grant APR 73-07840-A02, "Development of an Improved Transportation and Land Use Model Package."

A large portion of the computer work necessary to produce these results was done by Fred Ducca, a graduate student in the Department. Earlier work on fitting portions of the models was done by Chuck Sawyer and Rob Mathie, both graduate students in the Department.

Introduction

As part of a National Science Foundation sponsored effort to compare the performances of two different land use models calibrated on the same data base, several fundamental problems in urban land use modelling have been encountered and partially resolved. In particular, the fact that no Lowry derivative land use model had ever been properly calibrated in U.S. practice became abundantly clear. In order then, to accomplish the desired comparison of different models on a common data base, it became necessary to develop a calibration procedure for these models. The development of this calibration procedure in turn suggested a reformulation of the model which appears to be much superior to the original and which is sufficiently different to justify a new name, Disaggregated Residential Allocation Model - DRAM, which will differentiate it from its predecessor. A rather unique characteristic of this model, cast in entropy maximizing form, is its multivariate attractiveness measure.

Background

The development of the Lowry model of land use distribution (Lowry, 1964) along with that of numerous derivatives of its basic model structure has been described elsewhere (Goldner, 1971; Putman, 1975). Some years after development of these models had begun in the U.S., substantial further development of them was undertaken in Great Britain (Batty, 1972). Interestingly, despite the model's originating in the U.S., some of the most fruitful work in extending the concept has been done in recent years in Great Britain. Further, and of critical importance to applications of the model, the question of estimation of the model's parameters has to the knowledge of this author, never, with perhaps one exception (Voorhees, 1972) been properly settled in any U.S. work. In contradistinction, it appears that the British work has produced rather conclusive evidence as to the means by which these models may be calibrated (Batty, 1970; Batty and Mackie, 1972).

A modified version of the Incremental Projective Land Use Model (IPLUM) was used in the Integrated Transportation and Land Use Package (ITLUP). This ITLUP version of IPLUM is fully described elsewhere (Putman, 1973). In brief, the residential portion of this model allocates increments of residential locators to their places of residence in response to increments in basic employment and changes in the transportation facilities. This response is determined by a probability function which describes the distribution of work trips, and by a measure of residential attractiveness for each potential location zone. The purpose of this paper is to outline the steps thought to be necessary for a proper calibration of the ITLUP-IPLUM, and which ultimately led to the development of DRAM.

Virtually all Lowry derivative models used in the U.S. have as their residential allocation function some form of the following expression.

$$N_i = g \sum_j p_{ij} E_j \tag{1}$$

where,

N_i = number of residential locators locating in area i

p_{ij} = the probability of living in area i and working in area j

E_j = the number of employees in area j

g = a scaling factor such that the sum of the N_i over all i equals an exogenous control total there are often other scaling or multiplier factors to convert from employees to households and to assure internal consistencies of various types.

The p_{ij} is most important component of equation (1). In the original Lowry model, the function used was:

$$p_{ij} = (D_{ij})^{-1.33R} \tag{2}$$

where,

D_{ij} = airline distance between the centroids of area i and area j

R = number of zones in an annulus D_{ij} miles from the origin

In various of the Lowry derivative models, p_{ij} is modified to include measures of the attractiveness of area i. In particular in the ITLUP form of IPLUM,

$$p_{ij} = f(D_{ij})O_i \tag{3}$$

where,

$$f(D_{ij}) = (\beta/D^2_{ij}) \quad \exp(\alpha-\beta/D_{ij}) \tag{4}$$

O_i = a measure of residential "opportunities" in i

D_{ij} = travel time between centroids of zones i and j

α,β = empirically derived parameters

The measure of opportunities is basically an adjusted measure of residential holding capacity (previous level of residential density times amount of available land). The adjustment Q_i is a logistic curve function of, the proportion of the developable land in zone i which has been developed by the end of the base time period.

This is:

$$O_i = a_i^v(h_i/a_i^r)(Q_i) \qquad (5)$$

where

a_i^v = vacant acreage in zone i

h_i = housing units in zone i

a_i^r = residential acreage in zone i

Q_i = development level factor

and, where

$$Q_i = 1 - \frac{\gamma}{(1-\gamma)\exp(\delta x_i^2)} \qquad (6)$$

where

γ, δ = parameters

x_i = the percentage of developable land area in zone i which has been developed

The parameters of the trip function were estimated by fitting the equation to observed work-trip distributions from the San Francisco area. The parameters of the development level function Q_i have not been statistically estimated nor has the complete p_{ij} function been fit to any actual data. It was precisely this fitting of the complete p_{ij} function which was necessary, but which had never been done (excepting the Voorhees attempt) during U.S. work with Lowry model derivatives.

Reformulation of the Model

In all of these models the essence of the residential allocations is either the work-trip (home-to-work or work-to-home) or a combination of the work-trip with measures of attractiveness of the potential residential locations. Implicit therefore, in any of these models' estimates of residential locations, is a set of work trip estimates as well. Very little use has been made of this fact in U.S. practice. Yet, it is precisely the fact of these implicit trip matrices that leads to a more satisfactory method of estimating these models' parameters. The use of IPLUM in the ITLUP package is a particular exception to the usual ignoring of these implicit trips. In this case these implicit work trips are made explicit by extraction of the trips from the model directly. These trips are later used to load the transport network (Putman, 1973).

It is a virtue (and perhaps in the first instance was the source) of the Wilson entropy maximizing approach to analysis of these models that the question of these trips is made explicit (Wilson, 1967). For example, the Lowry model may be rewritten, based on this approach as (Wilson, 1970),

$$T_{ij} = E_j f(C_{ij}) \tag{7}$$

where

T_{ij} = number of persons working in zone j and residing in zone i

E_j = number of persons working in zone j

C_{ij} = impedence (usually travel time or travel cost) between centroids of zone i and zone j.

An important problem of this formulation is that there is no constraint on the sums of trips. Without the constraint there is no reason to expect that,

$$\sum_i T_{ij} = E_j \qquad (8)$$

This implies that the number of employees in zone j will not necessarily equal the sum of the employees residing in all zones i who claim to work in zone j.

A simple residential location model may be derived from entropy maximizing concepts as follows,

$$T_{ij} = A_i B_j O_i E_j f(C_{ij}) \qquad (9)$$

where

T_{ij}	=	trips between zones i and j or, number of persons living in zone i and working in zone j
O_i	=	trip origins or, employed persons living in zone i
E_j	=	trip destinations or, employees employed in zone j
A_i	=	balancing factor for trip origins
B_j	=	balancing factor for trip destinations
(C_{ij})	=	impedance function

It is possible to replace the trip origins O_i by a measure of attractiveness of the origin zone, W_i.. This eliminates the need for the origins balancing factor A_i thus giving

$$T_{ij} = B_j W_i E_j f(C_{ij}) \qquad (10)$$

In order for the constraint on the sums of trip destinations, equation (8), to be met, we have

$$B_j = \frac{1}{\sum_i W_i f(C_{ij})} \qquad (11)$$

It is informative to substitute this expression back into the original equation (Senior, 1973), which yields

$$T_{ij} = E_j \left[\frac{W_i f(C_{ij})}{\sum_i W_i f(C_{ij})} \right] \quad (12)$$

If the term $W_i f(C_{ij})$ is called an "accessibility attractiveness" measure, then the fraction in equation (12) is a relative measure of the accessibility-attractiveness of zone i to zone j compared to all other zones i. Further, it is clear that the total number of employed residents residing in zone i is

$$N_i = \sum_j T_{ij} \quad (13)$$

and, substituting

$$N_i = \sum_j \left\{ E_j \left[\frac{W_i f(C_{ij})}{\sum_i W_i f(C_{ij})} \right] \right\} \quad (14)$$

If one is willing to assert that,

$$p_{ij} = W_i f(C_{ij}) / \sum_i W_i f(C_{ij})$$

then equation (14) is equivalent to saying

$$N_i = \sum_j E_j p_{ij} \quad (15)$$

which is the same function as the Lowry model, described in equation (1).

Thus it can be seen that the IPLUM allocation procedure may be considered, in the context of the entropy maximizing formulation, as a simple residential location model. However, IPLUM is a dynamic model in that it estimates changes in the number of residential locators, as follows:

$$\Delta N_i = \sum_j (\Delta E_j)\, p_{ij} \quad (16)$$

where

ΔN_i = change in the number of employed residents of zone i from time to time t+1

ΔE_j = change in the number of employees in zone j from time t to time t+1

P_{ij} = probability that a person will live in zone i and work in zone j, at time t+1

A question arises here as to whether ΔP_{ij} might be more appropriate in the new formulation than P_{ij}? Resolution of this question leads unfortunately to the question, among others, of location of in-migrants versus location of intra-metropolitan movers. In-migrants probably make their location decisions somewhat differently than the intra-metropolitan movers. None of Lowry class of models deals properly with this question. The TOMM models (Crecine, 1964, 1969) do so in a very superficial way by means of the "stable-household" functions. It was not possible to resolve this problem in the current work, so the existing practice of using P_{ij} has been maintained for the present. Further, as will be discussed below, ultimately it was the static form of the model which was estimated.

Calibration: Initial Discussion

To date, virtually all U.S. attempts to calibrate these models have involved assorted procedures, no one of which achieved any more than a partial calibration of the allocation function. Some procedures have fitted an $f(D_{ij})$ as in equation (2) or equation (4) to observed trip data, without taking into account the effects of the characteristics of the origin zone or destination zone. Other calibration attempts have fit a function with N_i as the dependent variable and various characteristics of zone i as independent variables, thus ignoring any explicit consideration of the trip distribution. Neither of these two procedures nor any of their many variations is capable of properly estimating the parameters of such a model.

For a model expressed in the form of equation (9), the only parameter(s) to be estimated is/are the parameter(s) which may be included in $f(C_{ij})$. It has been shown that in the fitting of parameters for such a model, statistics summarizing the goodness of fit of the work trip distributions were much more sensitive to changes in model parameters than statistics summarizing the goodness of fit of the activity distributions (Batty, 1970). This result argues for the use when possible, of work trip statistics as criteria for model calibration. Other work has derived several summary statistics of the work trip distributions, each of which is appropriate for particular functional forms of $f(C_{ij})$ (Hyman, 1969).

A problem posed by the form of the model shown in equation (10) is that W_i, the attractiveness measure, is not a directly observable or measurable variable. In one model effort, number of dwellings in zone i or population in zone i were proposed as proxy measures of W_i (Cripps and Foot, 1969). Population was finally selected and produced quite acceptable calibration results. In another model effort, usable land area in zone i were suggested as proxy measures of W_i (Barras, et. al., 1971). In both of these cases, by using a single proxy variable for W_i, calibration of the model remains as a matter of estimating the parameter(s) of $f(C_{ij})$.

In these cases, as well as those using the original form of the model in equation (9), the calibration process involves; a) selecting starting values of the parameters, b) estimating the trip distribution, c) comparing the estimated trip distribution to the actual trip distribution, d) revising the parameter values, and e) iterating to find the best fit parameter values. Work has been done on efficient means of doing this (Hyman, 1969; Batty and Mackie, 1972).

At this point, regretfully, it becomes necessary to introduce a troublesome consideration, the need to disaggregate the residential locators into types. First we acknowledge that this disaggregation may easily be described in terms of the entropy maximizing approach, by considering T_{ij}^{kw} to be the number of employees of type w who work in zone j and live in type k housing in zone i. An appropriate set of equations and constraints can be developed to cover this situation as well as several others (Wilson, 1970). Solving such a model involves an endogenous procedure for estimating the housing stock by zone. This is not a welcome prospect for our current research efforts though clearly it is a consideration for the future. What is necessary then is a model of the form of equation (10), but disaggregated only by type of locator. This may be written

$$T_{ij}^{k} = B_j^k E_j W_i^k f_k (C_{ij}) \tag{17}$$

then

$$B_j^k = \frac{1}{\sum_i W_i^k f_k(C_{ij})} \tag{18}$$

Finally, it seemed desireable to investigate the use of a multivariate attractiveness measure. There is empirical evidence that the attractiveness of zone i is a function of, among other vaiables, the distribution of household types living in zone i (Putman, 1973). This evidence suggests that the attractiveness of a zone to a particular household type is a function of the zone's percentage composition of household types. Further, the amount of developable land in a zone seems to be a determining factor in residential location, as does a developability factor which appears to act as a proxy variable for the extent of the available urban infra-structure. Thus a W_i^k may be defined as follows:

$$W_i^k = \left[\sum_g a_g^k (N_{ig} / \sum_g N_{ig})\right] r_i V_i Q_i \qquad (19)$$

where

a_g^k = parameters to be estimated

N_{ig} = number of households of type g in zone i, note the g household types correspond directly to the k household types

r_i = residential density - households/acre in zone i

V_i = available, developable, vacant land in zone i

Q_i = development level factor - see equation (6)

The parameters in the expression for Q_i may be estimated independent of the rest of the model. The parameters a_g^k need to be estimated within the structure of the model. In addition, the parameter(s) of the $f_k(C_{ij})$ must also be estimated within the structure of the model.

The precise form of the model desired would be, as per all the previous discussion, dynamic rather than static,

$$\Delta T_{ij}^k = B_j^k W_i^k (\Delta E_j^k) f_k (C_{ij}) \qquad (20)$$

To do this it would be necessary to have data for ΔT_{ij}^k and ΔE_j^k. At the time when this work was being done, these data were not available, making it impossible to estimate any but the static model.

In order to specify data requirements it will be helpful to write out the model in full.

$$T_{ijt}^k = B_j^k W_{it}^k E_{jt}^k f_k(C_{ijt}) \qquad (21)$$

Substituting in for B_j^k and W_i^k

$$T^k_{ijt} = E^k_{jt} \left[\frac{\sum_g a^k_g [N_{igt} / \sum_g N_{igt}] r_{it} V_{it} Q_{it} f_k(C_{ijt})}{\sum_i \left\{ \sum_g a^k_g [N_{igt} / \sum_g N_{igt}] \right\} r_{it} V_{it} Q_{it} f_k(C_{ijt})} \right] \tag{23}$$

Thus the required data are

T^k_{ijt} = the number of persons of type k employed in area j and living in area i at time t

E^k_{jt} = the number of persons of type k employed in zone j at time t

N_{igt} = the number of households of type g living in zone i at time t

r_{it} = residential density (households/acre) in zone i at time t

V_{it} = vacant developable land in zone i at time t

Q_{it} = development index, as described above, for zone i

C_{ijt} = travel cost (impedance) between the centroids of zones i and j at time t

Before discussing the calibration results, the perspicacious reader may have noticed a further problem, which exists with the definitions of T^k_{ijt}, E^k_{jt}, and N_{igt}. The E^k_{jt} are defined as number of persons of type k working in zone j at time t, and the T^k_{ijt} are number of persons of type k employed in area j and living in area i at time t. The N_{igt} however are number of households of type g living in zone i at time t. Clearly a conversion from employees to households is necessary at some point in the process. In order to simplify conversion of the T^k_{ijt} to vehicle trips for use in the network model, it will be most convenient to make the conversion at the residence end of the trip. Thus a matrix for converting households of type g to employees of type k must be developed from regional data for the regions to which the model is being fit. This was done for both the San Francisco and the Minneapolis - St. Paul applications, but the use of thèse regional conversion rates accross the board, makes it necessary to keep careful track of this conversion throughout the calibration process.

Calibration Results: Partial Estimates

It was initially intended that before the complete model equation was fit, preliminary estimates of its parameters would be developed by partial estimation of them by least squares regression. This was later found to be unnecessary, but some of the results related to the independent fitting of the trip distribution are of some interest.

It will be recalled that in equations (2) and (4) above, the distance functions used in the Lowry Model and in PLUM were given. These are but two of a vast number of functions which could be fitted to tripmaking data. To test several of these, a tabulation of the first work trips from the San Francisco Home Interview data file was prepared. These trips were tabulated according to the household and employment classes enumerated above for the 291 zone areal system. The distributions were then normalized and the resulting distributions were fit, using a non-linear least squares procedure, to several different functions. The work trip distributions took the familiar form shown in Figure 1.

Of the various functions investigated, the several varieties of gamma distribution seemed to produce the best fits. The general form of this

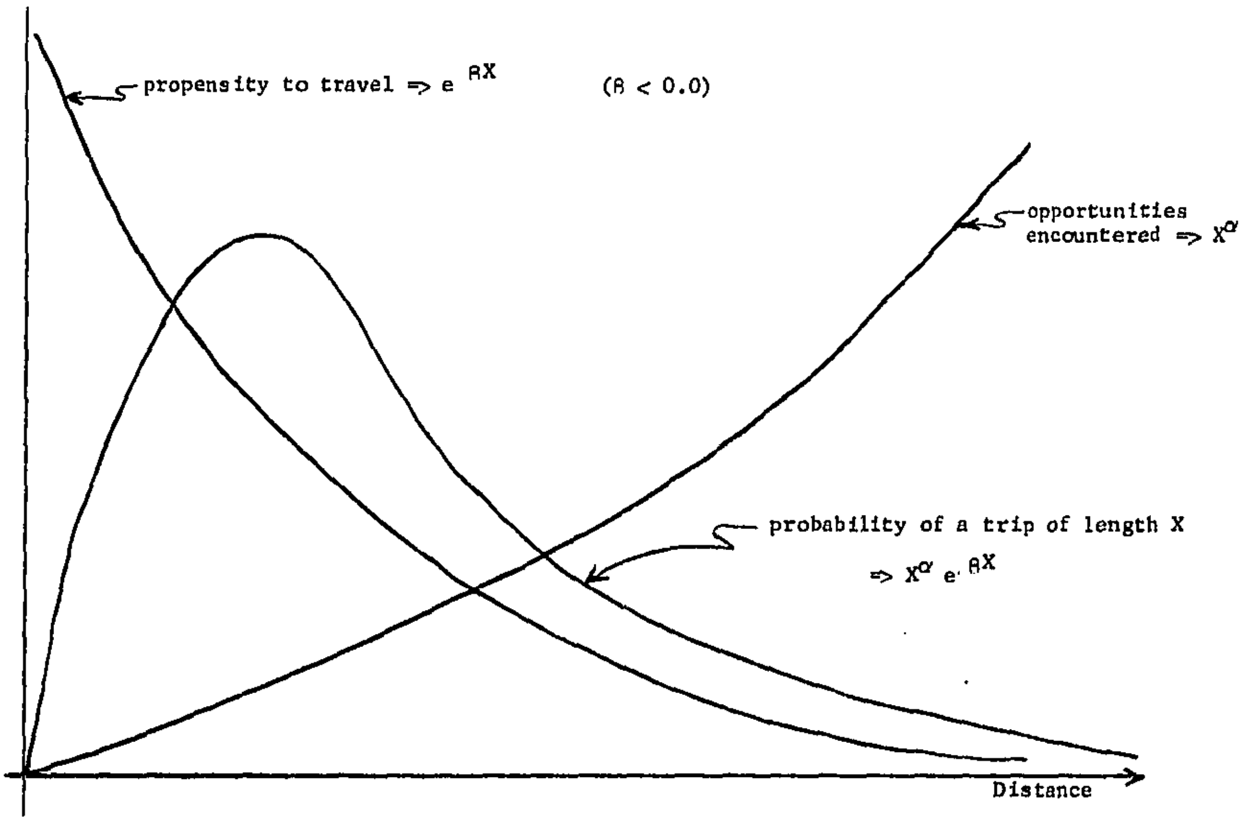

Figure 1: Trip Distribution Formulation

distribution is:

$$y = x^{\alpha} \exp\left\{f(x)\right\} \tag{23}$$

where

y = number of trips, or trip frequency

x = trip time or cost

The specific functions which best fit the data were sometimes best in one household income class and sometimes best in another. No one function was best for all four income classes. The function selected for further work on this prototype effort was

$$y = x^{\alpha} \exp(-\beta x) \tag{24}$$

This function, known as Tanner's function, had been used in this type model elsewhere (Cripps and Foot, 1969). The best fit parameters for the 291 zone system in San Francisco are shown in Table 1. These parameters in Tanner's function do yield the skewed, peaked, curve shown above.

In the calibrations described below, the San Francisco data were aggregated to a 30 zone system, thus increasing to greater than eight minutes the three minute average travel time between adjacent zones of the 291 zone system. At that scale all the values of α become negative and Tanner's function takes on the appearance of a simple declining exponential function. For the Minneapolis - St. Paul calibration (about 100 zones) the level of disaggregation is sufficient for the α to be positive again and for the skewed peaked curve to reappear. All of this reenforces the

Table 1: San Francisco - Work Trip Function Parameters

Income Class	α	β
$ 0 - 4999	0.383	0.900
5000 - 9999	0.750	0.963
10000 - 14999	0.849	0.992
> 15000	0.784	0.990

proposition that the level of spatial aggregation or disaggregation has noticeable effects on the apparent functional forms of these models.

Calibration Results: Complete Estimates for San Francisco

The preliminary estimates of parts of the model were of very little use except that they indicated that a product formulation for W would probably yield better fits than the sum form initially proposed. Consequently, equation (22) was rewritten. First, let

$$W_{it}^{k} = \left[\prod_{g} \left(N_{it}^{g} / \sum_{g} N_{it}^{g}\right)^{a_g^k}\right]\left[\prod_{m} \left(X_{it}^{m}\right)^{a_m^k}\right] \qquad (25)$$

where

N_i^g = the number of type g households in area i

X_i^m = a measure of attribute m of zone i

It was hoped that the attractiveness measure would continue to include intrinsic neighborhood attractiveness as indicated by the household types located there; a measure of "capacity" for development; and a measure of developability in terms of infrastructure. Various attributes were tested, including: residential density, vacant developable land, percentage of developable land developed, and percent industrial (basic) land.

The variables ultimately selected were:

n_i^k = the percentage of the total households in zone i which are of type k

V_i = available developable land in zone i

P_i = percentage of developable land in zone i which has been developed

r_i = residential density (households/acre) in zone i

Thus the form of W used in the final calibrations was (using four household types)

$$W^k_{i,t} = \left[\prod_{g=1}^{4} n_i^g \, (\exp a_g^k) \right] V_i^{a_5} \; P_i^{a_6} \; r_i^{a_7} \qquad (26)$$

Note that based on the preliminary estimates it was decided to replace the development level factor Q_i by a simple measure of existing level of development, P_i.

Then, rewriting equation (23) we get

$$T^k_{ijt} = E^k_{jt} \left[\frac{W^k_{it} \, f(C_{ijt})}{\sum_i W^k_{it} \, f(C_{ijt})} \right] \qquad (27)$$

Now there are two ways in which the parameters may be estimated. First, the simplest case, is by looking at the activities distribution(s). In this case, by definition:

$$\sum_j T^k_{ijt} = \text{number of households of class } k \text{ living in } i$$

and thus

$$N^k_{it} = \sum_j E^k_{jt} \left[\frac{W^k_{it} \, f(C_{ijt})}{\sum_i W^k_{it} \, f(C_{ijt})} \right] \qquad (28)$$

Consequently it is possible to estimate the parameters in the W and $f(C)$ functions and this may be called calibration of the aggregated form of the model.

Various authors have, however, asserted that there are disadvantages to calibration of the aggregated form of the model. Their remedy for these problems involves calibration of the disaggregated form of the model given in equation (27). It is an unfortunate fact that in order to calibrate the disaggregated form of the model it is necessary to have a good data source for the T's. In the work described here there were questions as to the quality of these data. If, at some later date, these questions can be satisfactorily resolved along with the development of an acceptable expansion of the San Francisco "sample" to an estimate of the "population", then a calibration of the disaggregated form may be undertaken. In the meantime, calibration has been undertaken for the aggregated form of the model only.

It is immediately obvious that equation (28) cannot be fit to a data set by use of the traditional procedures of linear or even nonlinear multiple regression. In fact the only procedures available are those which, by some hopefully efficient procedure, search for the parameters which produce the best fit of the model to the data. One such procedure is that of gradient search. The use of gradient search involves the following steps:

a) definition of a criterion function to be maximized or minimized
b) definition of the partial derivatives of the criterion function with respect to each of the parameters
c) selection of a starting point (parameters) and calculation of the criterion and the derivatives, hence the gradient, at that point
d) alteration of the parameters as a function of the calculated derivatives and gradient, and iteration through steps c) and d) until a minimum or maximum has been reached

While this may sound like a rather lengthy and difficult undertaking, this is not actually the case. The computer software is somewhat difficult but is available from a variety of sources, including the University of Pennsylvania. It does, at this stage in its development, require experienced staff for its proper use. Nevertheless, once set up, the procedure is rather straightforward and results may be quickly obtained.

The San Francisco data were aggregated to a thirty zone areal system primarily for operating economy in the face of no prior experience as to the costs and difficulties of performing such calibrations. It was felt that the thirty zone system would take less computer time to calibrate while still providing useful information about both the model and the calibration process in general.

The model to be fit is given in equation (28). The distance function is that of equation (24). The variables in the attractiveness measure are the same as were used in equation (25). The calibration was achieved with surprisingly little difficulty. Once the programs were operating correctly there were no significant problems encountered. An interesting point is that a broad, flat ridge in n-space was found where the search program's criterion value, R^2, was somewhat insensitive to parameter variations. This was an expected occurrance, as suggested above (Batty, 1970), nonetheless, with patience, a maximum was reached. The parameters found are shown in Table 2.

There are a number observations to be made regarding these parameter values. Principally, before leaping to unwarranted conclusions, it must be remembered that the household data used in these runs is from the 1960 census, while the land use and employment data are from surveys conducted in San Francisco in 1965. Thus the time subscripts for these variables are not correct for the formulation of the model. The purpose of this particular effort was to explore the problems of calibration of Lowry derivative models via the Wilson entropy approach. That this is a practical procedure has been amply demonstrated.

Table 2: BEST FIT PARAMETERS (EXPONENTS) - DRAM - SAN FRANCISCO (30 ZONES)

	Household Composition				Land Development			Distance		
Household type	a_1^k	a_2^k	a_3^k	a_4^k	a_5	a_6	a_7	α	β	r^2
$ < 5000	1.90	0.40	-0.50	0.33	0.18	-0.73	-0.26	-2.06	0.57	0.91
$5000 - 10000	0.06	1.65	-1.22	0.48	0.27	-1.50	-0.07	-1.75	0.72	0.87
$10000 - 15000	0.14	1.09	-0.26	0.76	0.24	-1.34	-0.14	-1.76	0.76	0.90
$ > 15000	0.72	1.00	-0.34	1.50	0.23	-1.48	-0.04	-1.64	0.48	0.93

Calibration Results: Complete Estimates for Minneapolis - St. Paul

The DRAM model was also calibrated for an available data base for the Minneapolis - St. Paul metropolitan area. This area was divided into 108 zones. The equation form used was also that of equation (27) with the distance function of equation (23). The household income classes differed from those of San Francisco in that they were income quartiles. One of the attractiveness measures, r^i - residential density was replaced by R_i - residential land, which produced better fits. The results of these estimates are shown in Table 3.

The data used in this case are all from approximately 1970, thus resulting in parameter estimates for a static form of the model. It is interesting to note that the scaling or control total procedures, typically used in these models after the allocations are completed, have moved, with the DRAM reformulation, deeper into the workings of the model. Referring to equation (27) it may be seen that the term in brackets on the right-hand side is a proportion. Consequently each E^k_{jt} is simply allocated over all i zones. Consequently the sum of the N^k_i will be equal to the sum of the E^k_j. It was mentioned above that it was necessary to convert the E^k_j from employees of type k to heads of households of type k. If it is assumed that the E^k_j sum to a prespecified regional employment total (or are forced to do so) then the N^k_i can be forced to sum to a regional population total as part of the employee to head of household conversion. This, while still arbitrary, is not so arbitrary as the various forms of scaling procedure typically used in these models, which often involve altering sophisticated model estimates with rather crude prorating procedures and thus vitiating the model results.

Table 3: BEST FIT PARAMETERS (EXPONENTS) - DRAM - MINNEAPOLIS - ST. PAUL (108 ZONES)

	Household Composition				Land Development			Distance		
Household Type	a_1^k	a_2^k	a_3^k	a_4^k	a_5	a_6	a_7	α	β	r^2
First Quartile	0.77	0.14	-0.56	-0.34	-0.03	0.15	0.89	1.04	2.18	0.93
Second Quartile	0.24	0.84	-0.37	-0.15	-0.04	0.18	0.90	2.11	1.46	0.90
Third Quartile	0.09	0.16	0.50	-0.08	-0.08	0.25	0.80	2.81	1.31	0.90
Fourth Quartile	0.13	0.10	-0.19	0.78	-0.03	0.29	0.75	2.10	1.44	0.91

Discussion - Problems of Calibrations

The work described here was originally undertaken simply for the purpose of exploring the possibility of calibrating a Lowry-derivative model with a multivariate attractiveness measure via the Wilson entropy formulation. That this is possible has been amply demonstrated. Nonetheless, problems with the available data, particularly with respect to their time indices, makes interpretations of the substance of the results somewhat chancy.

The general question of parameter interpretation in models of this form is worth discussing. First note that the scale of any of the variables is immaterial since the effect of the balancing factor (equation 11) will be to normalize each variable in all cases. Thus parameters may be interpreted in terms of a variable which ranges from zero to one. Care must be taken to avoid having a variable reach zero if its exponent is negative and checks should be incorporated in both parameter estimation and forecasting programs to, at the least, alert the user to this situation if it should arise.

In Figure 2 several members of the family of curves of the form $y=x^{\alpha}$ are plotted for different values of α. The range in which we are particularly interested is from x=0 to x=1. Taking first the case of $\alpha \geq 0$, we see that for any α the value of y is < 1. Thus any variable x_i, for which the estimated α is < 1, will have an attenuating effect on the region's share of households in area i. This attenuation gradually diminishes as the value of x_i increases from zero to one. It is important to note that the intuitive expectation of a variable with a positive exponent being an amplifying variable is not quite correct here. In the case of a variable whose range is zero to one, a positive exponent implies decreasing attenuation with increases in the magnitude of the variable to its limit of one.

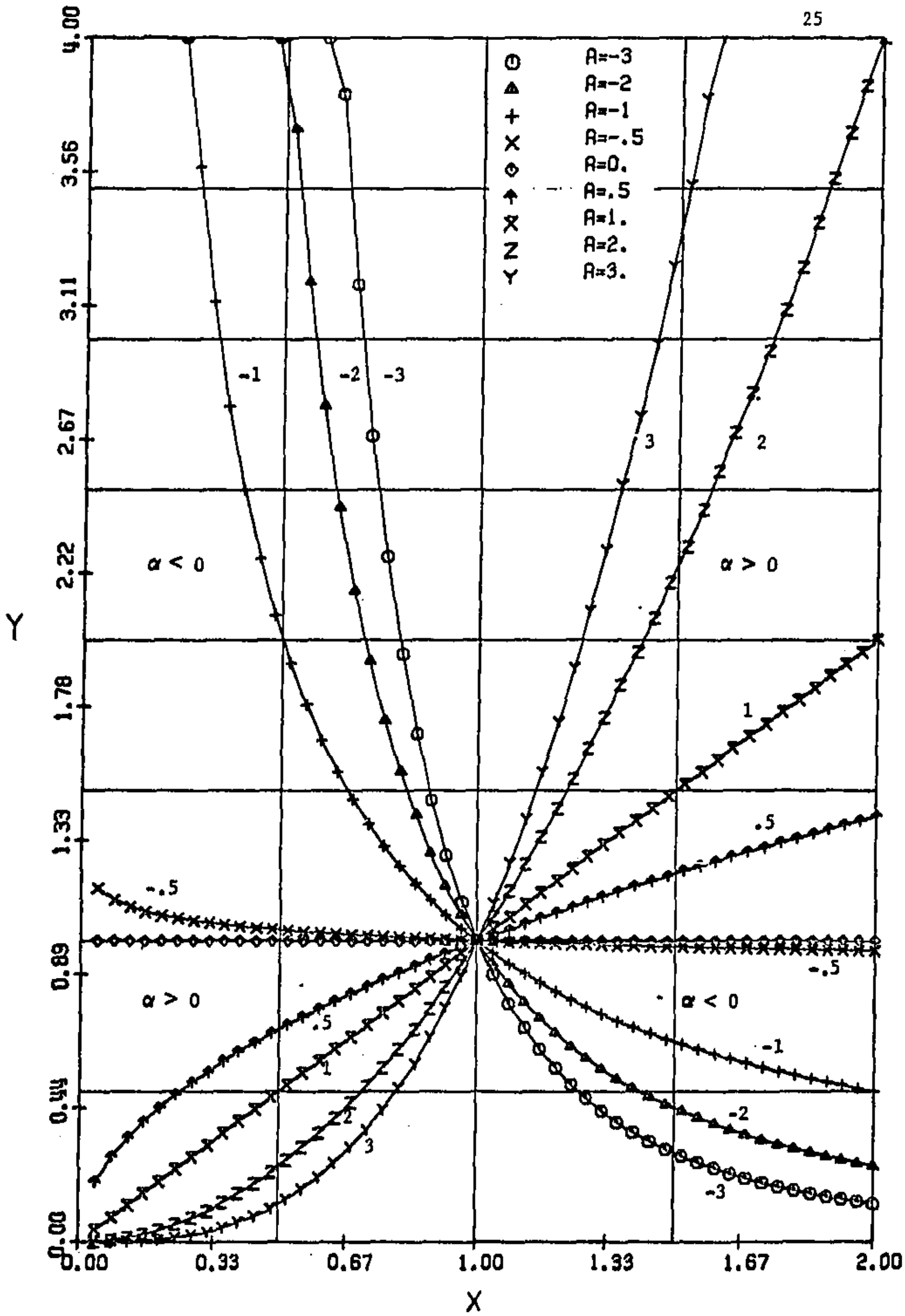

Figure 1: PLOTS OF Y=X expα FOR 0<X<2 AND α= -3,-2,-1,-.5,0,.5,1,2,3

The case of $\alpha < 0$ produces considerable amplification for very small values of x, with the amount of amplification decreasing as x increases to its limit of one. Again, the intuitive notion of a variable with a negative exponent being an attenuating variable is not quite correct. For the case of variables whose range is zero to one, a negative exponent implies decreasin amplification as the variable goes from zero to its limit of one.

In the static situation, thinking about each zone vis-a-vis all other zones makes more sense. For a variable with a positive value of α, all other variables being equal, one would expect greater values of the dependent variable to be found with greater values of the independent variable with a positive α. Similarly lesser values of the dependent variable would be expected to be found with greater values of the independent variable with a negative α. This reasoning also holds for the situation of increases or decreases in the particular independent variable. Nonetheless, it must be remembered that interpretation of the model's parameters does involve the notions of decreasing attenuation producing increases and decreasing amplification producing decreases, and that this is, to a certain degree counter-intuitive.

In this same connection the use of the exponential product form of the model caused some operating difficulties. These are when one or another of the independent variables approaches zero. It may easily be seen in Figure 1 that near zero the function y = x exp α becomes rather volatile for all non-zero values of α. Consequently the Minneapolis - St. Paul data were rerun with all the independent variables with ranges from 0.0 to 1.0 shifted to the range 1.0 to 2.0 by simply replacing, say P_i, by $(1.0 + P_i)$. These results are shown in Table 4. While there are some noticeable changes in the coefficients compared to the results in Table 3, the overall patterns

Table 4: REVISED BEST FIT PARAMETERS (EXPONENTS) - DRAM - MINNEAPOLIS - ST. PAUL (108 ZONES)

	Household Composition				Land Development			Distance		
Household Type	a_1^k	a_2^k	a_3^k	a_4^k	a_5	a_6	a_7	α	β	r^2
First Quartile	2.92	0.62	-1.71	-1.82	-0.10	0.55	0.83	0.92	2.14	0.89
Second Quartile	1.51	2.04	-1.36	-1.57	-0.06	0.65	0.85	2.24	1.36	0.88
Third Quartile	0.03	0.45	1.06	-0.64	-0.09	0.60	0.87	2.84	1.32	0.89
Fourth Quartile	-0.54	-0.55	-0.06	1.33	-0.07	0.63	0.88	2.48	1.52	0.86

of coefficients are virtually identical. In this form both the problems of instability as the variables approach zero, and of the counter-intuitive operation of the exponents are remedied.

It is very difficult to refrain from speculation as to the substantive implications of the parameters obtained in these estimations. Nonetheless, this would be the wisest policy at this time. We cannot, however, resist the temptation to call attention to the household composition variables and the interesting speculations which the reader may wish to draw therefrom. Two questions are posed here which should be explored during further work with the model. First, with regard to these parameters of the household composition in each zone, is there an apparent preference amongst household types for "equals" or "betters" i.e. higher income classes? Further, if this preference appears is it a preference for the amenities with with which they are associated? Second, having seen how a change in the size of the areal unit changes the shape of the travel function, one wonders at the effect of such a change on the attractiveness portion of the model. To the extent that the household compositions are representative measures of a complex of variables, their meaning may be lost on large areas. The representation, for example, of neighborhood which may show up at a small area level may disappear when the areas are aggregated to larger zones.

Another set of questions which must be resolved during further work with this model has to do with the interaction between the "travel parameters and the "attractiveness" parameters. In these experiments one might first constrain the attractiveness parameters to zero and observe the fit of data to the travel function only, within the construct of the model. Then the reverse could be explored by constraining the travel parameters to zero and observing the fit of data to the attractiveness function only. This information might have been obtained from the independent fitting of the two parts of the model formulation as described above. However, the functions used were not quite correct, nor were the data.

In retrospect it seems that the earlier independent estimation of portions of the model done for the San Francisco data was unnecessary in terms of estimating starting values of parameters for the complete model estimation. The knowledge obtained about the appropriate functional forms to be used in the complete model was, however, a worthwhile output. In future calibration work with this model it will probably be more efficient to begin with the complete model form, while perhaps omitting some of the attractiveness variables or at least constraining their parameter values for the first few runs while initial values of the other parameters are determined. This procedure seemed to work reasonably well for the Minneapolis - St. Paul data. Finally, it should be noted that the use of r^2 as the criterion for parameter fitting is not clearly the best criterion for functional forms like DRAM. The use of maximum likelihood criteria is being investigated for future work.

In conclusion, the initial tests of this model formulation are quite promising. The model appears to be capable of providing direct spatial allocations of households, by several types, without the need for complex input variables or involved sets of constraints and adjustments which are usually found at the tail-end of land use models. At the time of this writing an effort is underway to reevaluate much of the work described here and to produce a more final and definitive form and calibration of DRAM.

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Appendix II

Tables of Model Outputs

Table A1: TEST RESULTS OF DELETING BASE YEAR LOW INCOME HOUSEHOLDS FROM ZONE (#57) (ALLOWING THEIR RETURN)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	14	1123	- 98.8	368	368	+0.0
Low-Middle Income	52	138	- 62.3	0	0	+0.0
High-Middle Income	165	66	+150.0	0	0	+0.0
High Income	52	12	+333.3	0	0	+0.0
Total Population	283	1339	- 78.9	368	368	+0.0
Basic 1	4153	4153	+ 0.0	708	708	+0.0
Basic 2	1315	1315	+ 0.0	3627	3628	+0.0
Non-Basic	21714	12714	+ 0.0	3609	3610	+0.0
Total Employment	18183	18182	+ 0.0	7944	7946	+0.0

Table A2: TEST RESULTS OF DELETING BASE YEAR LOW INCOME HOUSEHOLDS FROM ZONE (#57) (NOT ALLOWING RETURN)

	Results of DRAM Package Runs		
Sector	Test Run	Control Run	Percent Difference
Low Income	0	682	** *
Low-Middle Income	98	213	- 54.0
High-Middle Income	112	87	+ 28.7
High Income	72	54	+ 33.3
Total Population	282	1036	- 72.8
Basic 1	4153	4153	0.0
Basic 2	1315	1315	0.0
Non-Basic	12725	12725	0.0
Total Employment	18193	18193	0.0

Table A3: TEST RESULTS OF DELETING BASE YEAR LOW INCOME HOUSEHOLDS FROM ZONE (#49)
(ALLOWING THEIR RETURN)

	Results of DRAM Package Runs		
Sector	Test Run	Control Run	Percent Difference
Low Income	928	1046	- 11.3
Low-Middle Income	2123	2377	- 10.7
High-Middle Income	3866	3908	- 1.1
High Income	4267	4071	+ 4.8
Total Population	11184	11402	- 1.9
Basic 1	3258	3258	0.0
Basic 2	667	667	0.0
Non-Basic	1981	1983	- 0.1
Total Employment	5906	5908	0.0

Table A4: TEST RESULTS OF DELETING BASE YEAR LOW INCOME HOUSEHOLDS FROM ZONE (#49)
(NOT ALLOWING RETURN)

	Results of DRAM Package Runs		
Sector	Test Run	Control Run	Percent Difference
Low Income	0	1046	- **
Low-Middle Income	1831	2377	- 22.9
High-Middle Income	3915	3908	- 0.2
High Income	4529	4071	+ 10.1
Total Population	10275	11402	- 9.9
Basic 1	3258	3258	0.0
Basic 2	667	667	0.0
Non-Basic	1974	1983	- 0.5
Total Employment	5899	5908	- 0.2

Table A5: TEST RESULTS OF DELETING BASE YEAR LOW INCOME HOUSEHOLDS FROM ZONE (#56) (ALLOWING THEIR RETURN)

Sector	Results of DRAM Package Runs		
	Test Run	Control Run	Percent Difference
Low Income	8884	9836	- 9.7
Low-Middle Income	4994	5342	- 6.5
High-Middle Income	3089	3080	+ 0.3
High Income	2852	2771	+ 2.9
Total Population	30887	30737	+ 0.5
Basic 1	1822	1822	0.0
Basic 2	1237	1237	0.0
Non-Basic	28543	28564	- 0.1
Total Employment	31602	31623	- 0.1

Table A6: TEST RESULTS OF DELETING BASE YEAR LOW INCOME HOUSEHOLDS AND ADDING 1000 UPPER INCOME HOUSEHOLDS TO ZONE (#56)

	Results of DRAM Package Runs		
Sector	Test Run	Control Run	Percent Difference
Low Income	8121	9836	- 17.4
Low-Middle Income	4617	5342	- 13.6
High-Middle Income	3040	3080	- 1.3
High Income	2978	2771	+ 7.5
Total Population	18756	21029	- 10.8
Basic 1	1822	1822	0.0
Basic 2	1237	1237	0.0
Non-Basic	28528	28564	- 0.1
Total Employment	31587	31623	- 0.1

Table A7: TEST RESULTS OF ADDING 1000 LOW INCOME HOUSEHOLDS TO ZONE 37 IN THE BASE YEAR

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	2174	1179	+ 84.4	795	797	+ 0.0
Low-Middle Income	2583	2058	+ 25.5	1756	1759	- 0.0
High-Middle Income	3030	3113	- 2.7	2210	2212	- 0.0
High Income	3175	3339	- 4.9	1254	1253	+ 0.0
Total Population	10962	9689	+ 13.1	6017	6019	- 0.0
Basic 1	687	687	+ 0.0	467	467	+ 0.0
Basic 2	228	228	+ 0.0	166	164	+ 1.2
Non-Basic	2696	2687	+ 0.3	3435	3436	- 0.0
Total Employment	3611	3602	+ 0.2	4068	4067	+ 0.0

Table A8: TEST RESULTS OF ADDING 1000 LOW INCOME HOUSEHOLDS TO ZONES 33 AND 32 IN THE BASE YEAR

	Results of DRAM Package Runs (Zone 33)			Results of DRAM Package Runs (Zone 32)		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	1038	654	+ 58.7	816	518	+ 57.5
Low-Middle Income	2333	1594	+ 46.4	2019	1381	+ 46.2
High-Middle Income	2302	2284	+ 0.8	2048	2041	+ 0.3
High Income	1746	2028	- 13.9	1536	1774	- 13.4
Total Population	7419	6560	+ 13.1	6419	5714	+ 12.3
Basic 1	968	968	0.0	1379	1379	0.0
Basic 2	213	213	0.0	421	421	0.0
Non-Basic	1728	1719	+ 0.5	671	668	+ 0.4
Total Employment	2909	2900	+ 0.3	2471	2468	+ 0.1

Table A9: TEST RESULTS OF ADDING 1000 LOW INCOME HOUSEHOLDS TO ZONE 94 IN THE BASE YEAR

	Results of DRAM Package Runs		
Sector	Test Run	Control Run	Percent Difference
Low Income	326	134	+143.0
Low-Middle Income	472	354	+ 33.3
High-Middle Income	560	616	- 9.1
High Income	172	212	- 18.9
Total Population	1530	1316	+ 16.3
Basic 1	840	840	+ 0.0
Basic 2	160	160	+ 0.0
Non-Basic	374	373	+ 0.3
Total Employment	1374	1373	+ 0.1

Table A10: TEST RESULTS OF ADDING 1000 HIGH INCOME HOUSEHOLDS TO ZONE 74 AND 1000 HIGH INCOME PLUS 1000 UPPER MIDDLE INCOME HOUSEHOLDS TO ZONE 94 IN THE BASE YEAR

	Results of DRAM Package Runs (Zone 74)			Results of DRAM Package Runs (Zone 94)		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	3596	3839	- 6.3	77	133	- 42.1
Low-Middle Income	3367	3605	- 6.6	222	405	- 45.2
High-Middle Income	4002	4168	- 4.0	508	551	- 7.8
High Income	6756	6407	+ 5.4	384	297	+ 29.3
Total Population	17721	18019	- 1.7	1191	1386	- 14.1
Basic 1	5789	5789	0.0	840	840	0.0
Basic 2	5180	5180	0.0	160	160	0.0
Non-Basic	2023	2022	0.0	374	376	- 0.5
Total Employment	18071	18075	0.0	1374	1376	- 0.1

Table A11: TEST RESULTS OF A 5% ACCESS IMPROVEMENT FOR AN URBAN FRINGE ZONE (#24)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	160	143	+ 11.9	385	386	- 0.3
Low-Middle Income	626	547	+ 14.4	600	601	- 0.2
High-Middle Income	739	672	+ 10.0	698	698	0.0
High Income	352	294	+ 19.7	650	649	0.2
Total Population	1877	1656	+ 13.3	2333	2334	0.0
Basic 1	1042	993	+ 4.9	272	271	+ 0.4
Basic 2	58	106	- 45.3	0	0	0.0
Non-Basic	152	139	+ 9.4	1072	1066	+ 0.6
Total Employment	1252	1238	+ 1.1	1344	1337	+ 0.5

Table A12: TEST RESULTS OF A 20% ACCESS IMPROVEMENT FOR AN URBAN FRINGE ZONE (#24)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	323	143	+125.9	383	386	- 0.8
Low-Middle Income	942	547	+ 72.2	599	601	- 0.3
High-Middle Income	928	672	+ 38.1	698	698	+ 0.0
High Income	588	294	+100.0	651	649	+ 0.3
Total Population	2781	1656	+ 67.9	2331	2334	- 0.1
Basic 1	1020	993	+ 2.7	273	271	+ 0.7
Basic 2	115	106	+ 8.5	23	0	+ ***
Non-Basic	202	139	+ 45.3	1091	1066	+ 2.3
Total Employment	1337	1238	+ 8.0	1387	1337	+ 3.7

Table A13: TEST RESULTS OF A 5% ACCESS IMPROVEMENT FOR AN URBAN CORE ZONE (#64)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	22726	18806	+ 20.8	10107	10307	- 1.9
Low-Middle Income	12113	11874	+ 2.0	9530	9545	- 0.2
High-Middle Income	5244	5485	- 4.4	5148	5035	+ 2.2
High Income	4127	4063	+ 1.6	2855	2682	+ 6.5
Total Population	44210	40228	+ 9.9	27640	27569	+ 0.3
Basic 1	1762	1090	+ 61.7	1936	1907	+ 1.5
Basic 2	3090	3685	- 16.1	4797	4578	+ 4.8
Non-Basic	24450	21798	+ 12.2	22502	22251	+ 1.1
Total Employment	29302	26573	+ 10.3	29235	28736	+ 1.7

Table A14: TEST RESULTS OF A 20% ACCESS IMPROVEMENT FOR AN URBAN CORE ZONE (#64)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	39741	18806	+111.3	9502	10307	- 7.8
Low-Middle Income	12419	11874	+ 4.6	9486	9545	- 0.6
High-Middle Income	4305	5485	- 21.5	5488	5035	+ 9.0
High Income	4130	4063	+ 1.6	3368	2682	+ 25.6
Total Population	60595	40228	+ 50.6	3474	27569	+ 1.0
Basic 1	3298	1090	+202.6	2026	1907	+ 6.2
Basic 2	1615	3685	- 56.2	5430.	4578	+ 18.6
Non-Basic	30454	21798	+ 39.7	23256	22251	+ 4.5
Total Employment	35367	26573	+ 33.1	30712	28736	+ 6.9

Table A15: TEST RESULTS OF A 10% BASIC EMPLOYMENT INCREASE IN AN URBAN FRINGE ZONE (#24)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	145	143	+ 1.4	386	386	+ 0.0
Low-Middle Income	547	547	+ 0.0	601	601	+ 0.0
High-Middle Income	672	672	+ 0.0	698	698	+ 0.0
High Income	294	294	+ 0.0	649	649	+ 0.0
Total Population	1658	1656	+ 0.1	2334	2334	+ 0.0
Basic 1	1031	993	+ 3.8	271	271	+ 0.0
Basic 2	109	106	+ 2.8	0.0	0.0	+ 0.0
Non-Basic	139	139	+ 0.0	1064.	1066.	- 0.2
Total Employment	1279	1238	+ 3.3	1335	1337	- 0.1

Table A16: TEST RESULTS OF A 30% BASIC EMPLOYMENT INCREASE IN AN URBAN FRINGE ZONE (#24)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	151	143	+ 5.6	386	386	+ 0.0
Low-Middle Income	549	547	+ 0.4	601	601	+ 0.0
High-Middle Income	672	672	+ 0.0	698	698	+ 0.0
High Income	294	294	+ 0.0	649	649	+ 0.0
Total Population	1666	1656	+ 0.6	2334	2334	+ 0.0
Basic 1	1106	993	+ 11.4	271	271	+ 0.0
Basic 2	114	106	+ 7.5	17	0	+ ***
Non-Basic	139	139	+ 0.0	1060	1066	- 0.6
Total Employment	1359	1238	+ 10.0	1348	1337	+ 0.8

Table A17: TEST RESULTS OF A 10% BASIC EMPLOYMENT INCREASE IN AN URBAN CORE ZONE (#64)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	18831	18806	+ 0.1	10307	10307	+ 0.0
Low-Middle Income	11873	11874	+ 0.0	9545	9545	+ 0.0
High-Middle Income	5481	5485	- 0.1	5035	5035	+ 0.0
High Income	4062	4063	+ 0.0	2682	2682	+ 0.0
Total Population	40247	40228	+ 0.0	27569	27569	+ 0.0
Basic 1	1238	1090	+ 13.6	1903	1907	- 0.2
Basic 2	4141	3685	+ 12.4	5236	4578	+ 14.4
Non-Basic	21809	21798	+ 0.1	22186	22251	- 0.3
Total Employment	27188	26573	+ 2.3	29325	28736	+ 2.0

Table A18: TEST RESULTS OF A 30% BASIC EMPLOYMENT INCREASE IN AN URBAN CORE ZONE (#64)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	18880	18806	+ 0.4	10307	10307	+ 0.0
Low-Middle Income	11870	11874	+ 0.0	9545	9545	+ 0.0
High-Middle Income	5473	5485	- 0.2	5035	5035	+ 0.0
High Income	4061	4063	- 0.0	2682	2682	+ 0.0
Total Population	40284	40228	+ 0.1	27569	27569	+ 0.0
Basic 1	1539	1090	+ 41.2	1896	1907	- 0.6
Basic 2	5041	3685	+ 36.8	6538	4578	+ 42.8
Non-Basic	21830	21798	+ 0.1	22059	22251	- 0.9
Total Employment	28410	26573	+ 6.9	30493	28736	+ 6.1

Table A19: TEST RESULTS OF A 10% NON-BASIC EMPLOYMENT INCREASE IN AN URBAN FRINGE ZONE (#24)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	145	143	+ 1.4	386	386	+ 0.0
Low-Middle Income	548	547	+ 0.2	601	601	+ 0.0
High-Middle Income	672	672	+ 0.0	698	698	+ 0.0
High Income	294	294	+ 0.0	649	649	+ 0.0
Total Population	1659	1656	+ 0.2	2334	2334	+ 0.0
Basic 1	993	993	+ 0.0	275	271	+ 1.5
Basic 2	106	106	+ 0.0	0	0	+ 0.0
Non-Basic	152	139	+ 9.4	835	1066	- 21.7
Total Employment	1251	1238	+ 1.1	1110	1337	- 17.0

Table A20: TEST RESULTS OF A 30% NON-BASIC EMPLOYMENT INCREASE IN AN URBAN FRINGE ZONE (#24)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	150	143	+ 4.9	386	386	+ 0.0
Low-Middle Income	549	547	+ 0.4	601	601	+ 0.0
High-Middle Income	671	672	- 0.0	698	698	+ 0.0
High Income	294	294	+ 0.0	649	649	+ 0.0
Total Population	1664	1656	+ 0.4	2334	2334	+ 0.0
Basic 1	993	993	+ 0.0	270	271	- 0.4
Basic 2	106	106	+ 0.0	12	0	***
Non-Basic	180	139	+ 29.5	1184	1066	+ 11.1
Total Employment	1279	1238	+ 3.3	1466	1337	+ 9.6

Table A21: TEST RESULTS OF A 10% NON-BASIC EMPLOYMENT INCREASE IN AN URBAN CORE ZONE (#64)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	18958	18806	+ 0.8	10307	10307	+ 0.0
Low-Middle Income	11867	11874	- 0.1	9545	9545	+ 0.0
High-Middle Income	5472	5485	- 0.2	5035	5035	+ 0.0
High Income	4056	4063	- 0.2	2682	2682	+ 0.0
Total Population	40353	40228	+ 0.3	27569	27569	+ 0.0
Basic 1	1090	1090	+ 0.0	1991	1907	+ 4.4
Basic 2	3685	3685	+ 0.0	1684	4578	- 63.2
Non-Basic	23866	21798	+ 9.5	16074	22251	- 27.8
Total Employment	28641	26573	+ 7.8	19749	28736	- 31.3

Table A22: TEST RESULTS OF A 30% NON-BASIC EMPLOYMENT INCREASE IN AN URBAN CORE ZONE (#64)

	Results of DRAM Package Runs			Results of EMPIRIC Package Runs		
Sector	Test Run	Control Run	Percent Difference	Test Run	Control Run	Percent Difference
Low Income	19258	18806	+ 2.4	10307	10307	+ 0.0
Low-Middle Income	11853	11874	- 0.2	9545	9545	+ 0.0
High-Middle Income	5445	5485	- 0.7	5035	5035	+ 0.0
High Income	4043	4063	- 0.5	2682	2682	+ 0.0
Total Population	40599	40228	+ 0.9	27569	27569	.0
Basic 1	1090	1090	+ 0.0	1864	1907	- 2.3
Basic 2	3685	3685	+ 0.0	5971	4578	+ 30.4
Non-Basic	27943	21798	+ 28.2	25289	22251	+ 13.7
Total Employment	32718	26573	+ 23.1	33124	28736	+ 15.3